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Noether's Theorems and Conservation Laws in MHD and CGL Plasmas

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This paper discusses conservation laws in magneto-hydrodynamic (MHD) [1,2,3] and anisotropic pressure Chew-Goldberger-Low (CGL) [1,4,5] plasmas. The non-canonical Hamiltonian MHD equations were obtained by [6,7] and by [8]. [4] obtained non-canonical Hamiltonian equations for CGL plasmas. Both MHD and CGL action principles were obtained by [1] using a Lagrangian fluid dynamics approach. Both the MHD and CGL systems admit the 10 parameter Galilean Lie group. These symmetries give conservation laws via Noether's first theorem, namely: (a) the space and time translation symmetries give the momentum and energy conservation laws; (b) the Galilean boost symmetries give the center of mass conservation laws and (c) the rotational symmetries about the x, y and z axes give the angular momentum conservation laws. Both MHD and CGL plasmas have cross helicity conservation laws. The cross helicity conservation law in MHD [2,3] is a local conservation law if the gas is barotropic. In the non-barotropic case, the conservation law is nonlocal and depends on the integral of the temperature T back along the Lagrangian fluid particle path. Analogous cross helicity conservation laws also hold for CGL plasmas[5]. These conservation laws are due to a fluid re-labeling symmetry. [9] and [10] show that the magnetic helicity conservation law is due to a gauge symmetry for the electromagnetic field. We use the Lagrangian map and both canonical and evolutionary symmetry operators in our formulation of Noether's theorem, which are used to derive conservation laws using: (a) the classical approach of [13] and [14,15], and also using a more modern approach by [11] and [12]. [16] has generalized Noether's theorem to obtain conservation laws for partial differential equations not necessarily described by variational principles. [17] study Noether's second theorem and Ertel's theorem. Infinite classes of Lie dragged invariants in MHD and in fluids were obtained by [18], [19], and [20]. [21] gives a topological interpretation of magnetic helicity and [22] gives an analogous interpretation of cross helicity, in terms of generalized Aharonov-Bohm effects.

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