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Development of MUSES code for nonlinear MHD simulations with locally divergence-free discontinuous Galerkin method

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Development of three-dimensional nonlinear magnetohydrodynamics (MHD) code is a worldwide trend in the fusion plasma community. For whole device modeling of fusion reactors, the use of unstructured grids is highly recommended to reduce computational costs. In QST, a three-dimensional nonlinear full-MHD code "MUSES" is being developed with discontinuous Galerkin (DG) method. The DG method is originally designed to solve advection equations with high spatial accuracy and numerical stability, so it is expected that the DG method is suit to MHD because the governing equations of MHD can be decomposed into several characteristic waves such as Alfven wave.

However, calculation of the MHD equation is more difficult than that of Navier–Stokes equation; the following solenoidal constraint must be preserved for numerical stability:

$$\nabla \cdot \boldsymbol{B} = 0,$$

where $\boldsymbol{B}$ is the magnetic field. If this constraint is violated, unphysical force by magnetic monopole drives numerical instabilities. In addition, the accuracy of mechanical equilibrium can be also a significant issue especially for fusion plasmas. The equilibrium of fusion plasmas is described by the following equation:

$$\nabla p = \boldsymbol{J} \times \boldsymbol{B},$$

where p is the thermal pressure, and $\boldsymbol{J}$ is the current density. Both pressure gradient and Lorentz force are much stronger than the fluctuations, but they are exactly cancelled out to establish a mechanical equilibrium. What is important is our interest is focused on dynamics of the fluctuations. If accuracy of the mechanical equilibrium is not enough, the numerical noise of the equilibrium can be greater than the interested physics. Therefore, the numerical schemes should be well designed so as to express the equilibrium accurately.

In this talk, two advanced numerical schemes for the DG method are proposed which are essential for our fusion plasma simulation. The first one is locally divergence-free DG method for generalized curvilinear coordinates. In this scheme, the magnetic field is expressed as a linear combination of basis function vector $\boldsymbol{\Psi}$ which satisfies the following divergence-free condition:

$$\nabla \cdot \boldsymbol{\Psi} = 0.$$

In this work, the locally divergence-free DG method is extended to generalized curvilinear coordinates, and is implemented to the torus simulation. The other is an equilibrium-preserving scheme. If the momentum and total energy are discretized by the same basis functions, it is impossible to realize the numerical equilibrium since the pressure gradient does not have enough information to equilibrate Grad–Shafranov equation. To mitigate this issue, higher-order basis functions are implemented to the total energy in the equilibrium-preserving scheme.

As a verification of the proposed schemes, an analytic equilibrium for torus geometry is set as an initial condition, and time development is calculated by the examined schemes. Note that a steady state is the exact solution of this test problem. Figure 1 shows that the conventional scheme cannot maintain the initial equilibrium because of a checkerboard instability. The equilibrium-preserving scheme well reproduces the initial equilibrium, so it is expected that the proposed scheme enables to calculate the dynamics of fusion plasmas without catastrophic numerical noise.

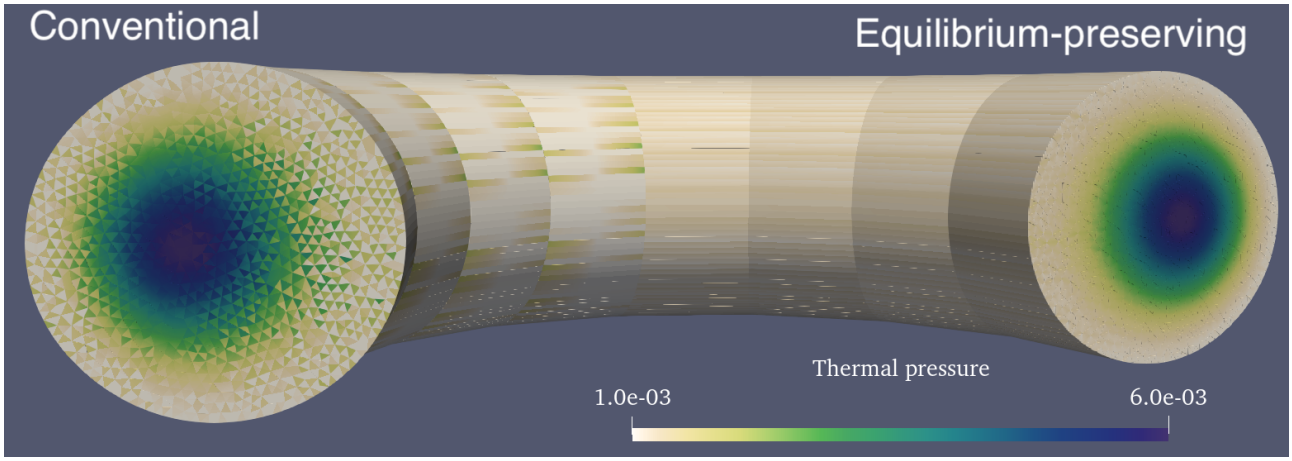

Figure 1. The conventional DG method is difficult to maintain an analytic equilibrium because of a checkerboard instability. Such a catastrophic instability can be mitigated with an appropriate selection of the basis functions.