

## Drift-kinetic perturbed Lagrangian for low-frequency nonideal MHD applications

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It is found that the perturbed Lagrangian derived from the drift-kinetic equation in the paper (Porcelli F et al 1994 Phys. Plasmas 1 470) [1] is inconsistent with the ordering for the low-frequency large-scale MHD. Here, we rederive the expression for the perturbed Lagrangian in the framework of nonideal MHD by using the ordering system for the low-frequency large-scale MHD in a low beta plasma. The obtained perturbed Lagrangian is consistent with Chen's gyrokinetic theory (Chen L and Zonca F 2016 Rev. Mod. Phys. 88 015008) [2], where the terms related to field curvature and gradient are small quantities of higher order and thus negligible. The main differences between Porcelli's results [1] and our new results [3] are summarized in Table 1. As the perturbed

Lagrangian has been widely used in literatures to calculate the plasma nonadiabatic response in low-frequency MHD applications, this finding may have significant impact on the understanding of the kinetic driving and dissipative mechanisms of MHD instabilities and the plasma response to electromagnetic perturbations in fusion plasmas.

### References

- [1] F. Porcelli et al, Phys. Plasmas **1**, 470 (1994)
- [2] L. Chen et al, Rev. Mod. Phys. **88**, 015008 (2016)
- [3] G. S. XU et al, Plasma Sci. Technol. **25**, 075104 (2023)

Table 1. The main different results of the derived perturbed Lagrangian.

	Previous results [1]	New results [3]
Total perturbed vector potential $\tilde{\mathbf{A}}$	$\tilde{\mathbf{A}} = \tilde{\mathbf{A}}_{\perp}$	$\tilde{\mathbf{A}} = \tilde{\mathbf{A}}_{\perp} + \tilde{\mathbf{A}}_{\parallel}$ , $ \tilde{\mathbf{A}}_{\parallel}  \gg  \tilde{\mathbf{A}}_{\perp} $
Parallel perturbed vector potential $\tilde{\mathbf{A}}_{\parallel}$	$\tilde{\mathbf{A}}_{\parallel} = 0$ , $\tilde{\mathbf{B}}_{\perp} = \nabla \times \tilde{\mathbf{A}}_{\parallel} = 0$	$\tilde{\mathbf{A}}_{\parallel} = \nabla_{\parallel} (\tilde{\varphi} - \tilde{\Psi}) / i\omega$ , $ \tilde{\mathbf{B}}_{\perp}  \gg  \tilde{\mathbf{B}}_{\parallel} $
Perpendicular perturbed vector potential $\tilde{\mathbf{A}}_{\perp}$	$\tilde{\mathbf{A}}_{\perp} = \tilde{\xi}_{\perp} \times \bar{\mathbf{B}}$ , $\tilde{\mathbf{B}}_{\parallel} = \nabla \times \tilde{\mathbf{A}}_{\perp}$	$\tilde{\mathbf{A}}_{\perp} = \tilde{\xi}_{\perp} \times \bar{\mathbf{B}} + \nabla_{\perp} \tilde{\varphi} / i\omega$  Two terms are largely canceled
Displacement of a field line $\tilde{\xi}_{\perp}$	$\tilde{\xi}_{\perp} = \bar{\mathbf{b}} \times \tilde{\mathbf{A}}_{\perp} / \bar{B}$  Only applicable to the high-frequency dynamics	$\tilde{\xi}_{\perp} = \bar{\mathbf{b}} \times (\tilde{\mathbf{A}}_{\perp} - \nabla_{\perp} \tilde{\varphi} / i\omega) / \bar{B}$  Applicable to the low-frequency dynamics
Derived perturbed Lagrangian $\tilde{L}$	$-(mv_{\parallel}^2 - \bar{\mu}\bar{B}) \tilde{\xi}_{\perp} \cdot \bar{\kappa} + \bar{\mu}\bar{B}\nabla \cdot \tilde{\xi}_{\perp}$  contained in $Ze\tilde{\mathbf{A}}_{\perp} \cdot \bar{\mathbf{v}}_B - \bar{\mu}\tilde{\mathbf{B}}_{\parallel}$ of our new expression	$Ze\tilde{\mathbf{A}}_{\parallel}v_{\parallel} - Ze\tilde{\varphi}$ $+ (Ze\tilde{\mathbf{A}}_{\perp} + mv_{\parallel}\tilde{\mathbf{B}}_{\perp}/\bar{B}) \cdot (\bar{\mathbf{v}}_E + \bar{\mathbf{v}}_B)$ $-\bar{\mu}(\tilde{\mathbf{B}}_{\parallel} + \tilde{\mathbf{B}}_{\perp}^2/2\bar{B})$