



Lagrange Multiplier Formulation of Ideal Magnetohydrodynamics (IMHD)

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In the standard formulation of IMHD the electric field is eliminated by taking the curl of the Ideal (i.e. zero resistivity) Ohm's Law (IOL) and using the Faraday-Maxwell induction equation. This builds in the "frozen-in flux" topological constraint [1] that forbids reconnection and hence magnetic island creation and destruction in magnetic toroidal confinement, which leads to singular behavior at rational magnetic surfaces in 3-D equilibria when regarded as the long-time limit of a damped evolution away from an initially axisymmetric state.

A recent paper [2] derives a new version of IMHD by enforcing (weakly or strongly) the IOL constraint using a Lagrange multiplier field that can be identified physically as a time-space-varying electrostatic polarization vector (cf. [3]). This affords the possibility of regularizing IMHD by relaxing the frozen-in flux constraint using a sequence of approximating Lagrange multipliers.

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References

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