

## Reduction of Anisotropy in Numerical Dispersion in the Explicit Finite-Difference Time-Domain Method with Laplacian

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The Finite-Difference Time-Domain (FDTD) method is a numerical method for solving the time development of electromagnetic fields.<sup>[1,2]</sup> The time-development equations used in the FDTD method is obtained by approximating Maxwell's equations with the finite difference of second-order accuracy in both time and space. A staggered grid (Yee grid) system is used in the spatial differences so that Gauss's law for both electric and magnetic fields is always satisfied. However, the FDTD method has a disadvantage that numerical oscillations occur in discontinuous waveforms and even in continuous waveforms with a large slope.

Higher-order finite differences reduce the numerical error in the phase velocity. The FDTD(2,4) method uses the fourth-order spatial difference.<sup>[3,4]</sup> The numerical phase velocity error of FDTD(2,4) is smaller than that of FDTD(2,2). However, the Courant condition of FDTD(2,4) method is more restricted than that of FDTD(2,2). In general, higher-order finite differences in space with the second-order finite difference in time make the Courant condition more restrictive, which requires smaller  $\Delta t$  and larger number of time steps.

Recently, FDTD method which relaxes the Courant condition has been developed.<sup>[5]</sup> The time-development equations of this method is derived by adding third-degree difference terms with coefficients to these of FDTD(2,4). However, there exists an anisotropy in the numerical dispersion, which results in numerical oscillations.

In the present study, a new explicit and non-dissipative FDTD method in two and three dimensions is proposed for reduction of anisotropy in numerical dispersion and relaxation of the Courant condition. By adding the third-degree spatial difference terms including Laplacian with coefficients to the time-development equations of

FDTD(2,4), a new schemes are developed.

Optimal coefficients are obtained by a brute-force search of the dispersion relations, which reduces phase velocity errors averaged over the entire wavenumber space but satisfies the numerical stability as well. The new scheme is stable with large Courant numbers up to  $C = 1$ . The new scheme also has smaller numerical errors in the phase velocity than conventional FDTD methods.

Figure 1 shows the results of numerical simulations in two dimensions. Panels (a) and (b) show the spatial profile of the magnetic field  $B_z$  contentment with Sekido23 and the present scheme with  $C = 1$ , respectively. Here, Sekido23 is the previous FDTD scheme which is proposed in the previous study.<sup>[5]</sup> Panel (a) shows wave propagation is depend on direction, and the numerical oscillations are the smallest at  $|x| = |y|$  with Sekido23. Panel (b) shows anisotropy in the waveforms is improved, and numerical oscillations are reduced with the present scheme.

### References

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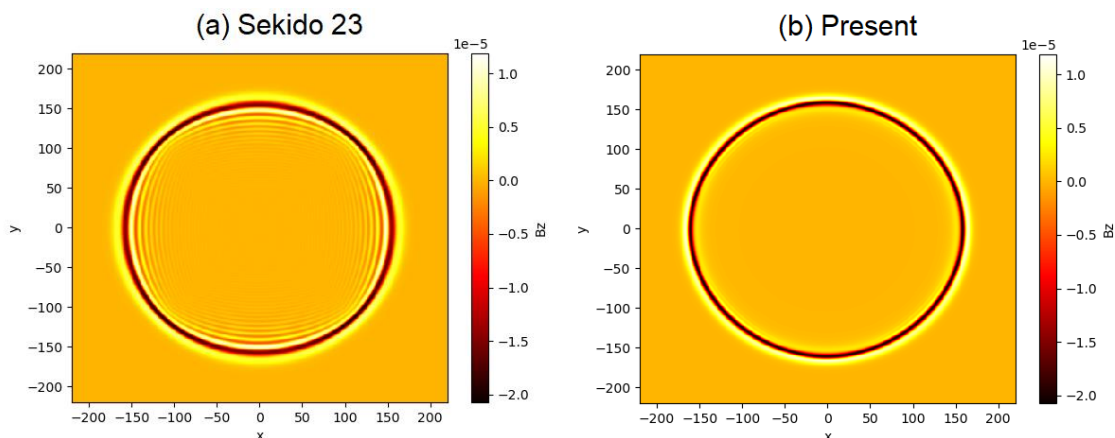


Figure 1. Spatial profiles of  $B_z$  in two dimensions with  $C = 1$ : (a) Sekido23; (b) present scheme.