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Study on Stability Analysis of Non-axisymmetric Tokamak Plasma Equilibrium by 3-D Multi-layers Method

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Recently, the importance of non-axisymmetric has been recognized in tokamaks. In order to consider the effect of non-axisymmetric in tokamaks in three dimensions, it is necessary to derive a three-dimensional MHD equilibrium that considers non-axisymmetric as a starting point. Currently, the most widely used 3D MHD balance code is the VMEC code,[1, 2] which uses the spectral method with a Fourier expansion for the angular variables over the magnetic surface. The VMEC code is presently used for 3D MHD balance calculations on most tokamaks. In contrast, VMEC uses magnetic coordinates, so it cannot be operated in the presence of a separatrix. VMEC also does not compute stability. The goal of this study is to obtain non-axisymmetric equilibrium with separatrix and to evaluate MHD stability by 3-D Multi-layers Method (MLM).

Like VMEC, MLM is an equilibrium analysis method based on the variational principle for finding extreme free energy values ($\delta U = 0$). MLM is distinguished from VMEC by the fact that MLM uses the XYZ coordinate system to compute the equilibrium with a separatrix, and by finding the minimum value of the free energy it is possible to find the equilibrium solution and find the stability at the same time.[3] The model consists of the plasma magnetic surface model, the external magnetic field coil model, and the conducting shell model (Figure 1.).

The plasma model is composed of multiple layers of magnetic surfaces (Ψ=const.), which coincides with the current surface in equilibrium, and the plasma current is assumed to be several ring-shaped line currents flowing only on the magnetic surface. The shape of the magnetic surface is defined by the Fourier series as follows.

$$R^k = \sum\nolimits_{mn} R^k_{mn} e^{i(m\theta - n\varphi)}, \tag{1}$$

$$Z^k = \sum\nolimits_{mn} Z^k_{mn} e^{i(m\theta - n\varphi)}, \tag{2}$$

where R, Z, k are the plasma magnetic surface coordinates and magnetic surface number.

The free energy of a line current system is given by

$$U(\boldsymbol{x}) = \frac{1}{2} \sum_{\Psi_i = \text{const.}} \Psi_i I_i(\boldsymbol{x}) - \frac{1}{2} \sum_{I_j = \text{const.}} \Psi_j(\boldsymbol{x}) I_j, \tag{3}$$

where Ψ_i, I_i , $\boldsymbol{x}$ are the linkage flux and current of the ith line current and the parameters determine the plasma position and shape. The first term represents the energy in the plasma, the vacuum vessel, and other parts that are not connected to the energy source. While the second term represents the contribution of the coil to which the feeder is connected. The magnetic flux Ψ_i and current $I_i$are related by

$$\Psi_i = \sum_j M_{ij}(\boldsymbol{x}) I_j, \tag{4}$$

where M_{ij} is inductance matrix. Using $\boldsymbol{x}$ as variables, we search for the minimum value of the free energy of the system by finding the mutual inductance while changing the shape of the plasma. The solution at the minimum is the stable equilibrium solution.

Using MLM, we analyzed the stability of the equilibria with longitudinal cross sections with no vertical positional instability by applying a non-axisymmetric magnetic field to the tokamak. This work is supported by the Grant-in-Aid for Scientific Research (21H01061).

References
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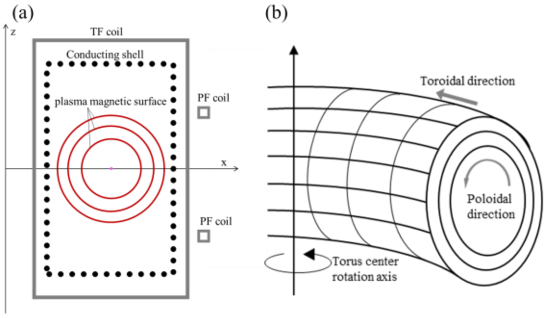

Figure 1. Multi-layers Method model. (a)Including plasma magnetic surface (red), conducting shell like vacuum vessel (black), and magnetic field coil (gray). (b) Plasma current is directed to the toroidal and poloidal rings on the magnetic surface. The conducting shell is treated as if the magnetic flux is constant, as is the plasma magnetic surface. Therefore, as the position and shape of the plasma change, the mutual inductance of the plasma between conducting shell also changes, as do the eddy currents.