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Band structure formation in rotating systems

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The problem of 2D turbulence in rotating systems has been intensively studied for a long time in relation to the formation and maintenance of zonal band structures and the predominant east-west jets observed on various planetary surfaces, such as Earth, Jupiter and Saturn.

The most fundamental model that have been considered is the β-plane model, which describes two-dimensional fluid motion on the tangent plane at a certain latitude on a rotating sphere. In this model, randomly forced many small-scale vortices evolve to larger vortices through the isotropic inverse cascade process. However, when the effect of rotation starts to dominate, the inverse cascade would stop and a zonal structure has been considered to emerge[1]. From the balance between the advection and β terms in the vorticity equation, the characteristic length scale $L_R = \sqrt{(U/\beta)}$, so called Rhines scale, can be obtained, where U and β are the characteristic amplitude of the flow and the background latitudinal potential vorticity gradient, respectively. Rhines scale is often used as an estimate of the order of magnitude of the width of the band structure appearing in two-dimensional turbulence of rotating systems.

In contrast, Vallis and Maltrud[2] performed numerical experiments on two-dimensional forced turbulence in a β plane model, and observed formation of large-scale zonal banded structures through the energy spectrum on a two-dimensional wavenumber space (k_x, k_y). Here k_x and k_y are the wavenumbers in the x (longitude) and y (latitude) directions, respectively. Figure 2 shows the 2D energy spectrum corresponding to Figure 1. Immediately after the start of the integration, the energy spectrum of the small-scale vortices produced by the forcing is distributed in a large circle (Fig. 2 left). After that, an isotropic inversion cascade to the origin is observed for large (k_x, k_y) region, whereas dumbbell-like blank regions are seen near the origin, which is produced by the β effect (Fig. 2 center and right). This means that the energy concentrates in the region along k_y-axis, avoiding the dumbbell region. Such a spectral distribution means that k_x is predominantly close to 0, and corresponding to the emergence of zonal banded structures.

They have shown that this dumbbell structure can be calculated with the oscillation frequency of Rossby waves and that in isotropic turbulence (reciprocal of the advection time of the turbulent eddies). For example, when the frequency of turbulence is estimated as $U|k|$, we obtain, $k_x = k_R (\cos\theta)^{3/2}$, $k_y = k_R \sin\theta\,(\cos\theta)^{1/2}$, where $k_R = 1/L_R$. The outer region of this dumbbell is dominated by the advection term, meaning that the inverse cascade is possible there. This analysis shows that the inverse cascade of turbulence is not stopped by the β effect, but it proceeds in an anisotropic manner. After the appearance of the zonal structure due to the inverse cascade in the x direction, the inverse cascade can proceed slowly in the y direction. This anisotropic inverse cascade in the y direction, although time-consuming, is not inhibited until the spatial minimum wavenumber is reached. Therefore, a zonal banded structure with a width of the spatial maximum scale is considered to emerge eventually.

Merger and disappearance of the narrow jets that emerge at intermediate stages are observed in the forced turbulence problem on rotating spheres[3] and zonal flows driven by thermal convection in rapidly rotating thin spherical shells[4]. These phenomena would be caused by the similar mechanism to that operating in the β-plane model. On the other hand, such the anisotropic inverse cascade can be inhibited by other factors such as Ekman friction, which may cause narrow banded structures[5].

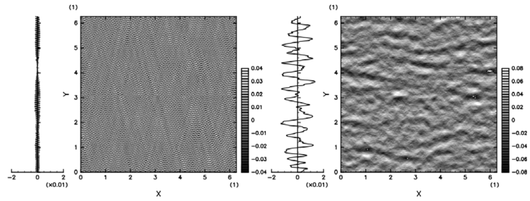

Figure 1. An example calculation for two-dimensional forced turbulence in the β plane. Zonal mean flows and velocity component in the x direction are shown. Light and dark areas represent positive and negative velocity components, respectively. Randomly forced many small-scale vortices (left) evolve to the zonal banded structure after time integration (right).

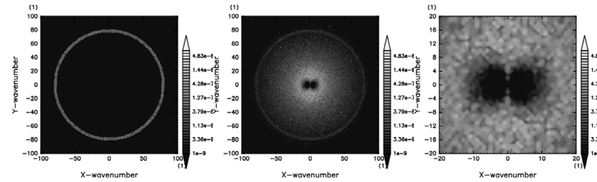

Figure 2. Two-dimensional energy spectrum in an example calculation of two-dimensional forced turbulence in the β plane. The left panel shows the spectrum for the state immediately after the start of time integration. The center panel is the spectrum for the state with the zonal banded structure. The right panel is the close up around the origin of the center panel.

References
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