

Recent advances in numerical schemes for plasma particle-in-cell simulations

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The particle-in-cell method is first established for plasma simulations, and has now become widely used in various scientific fields. A plasma particle-in-cell simulation code consists of three kernels. The first is the Maxwell equations for electromagnetic fields, the second is the equation of motions for charged particles, and the third is the continuity equation for charge. In this paper, recent advances in numerical schemes for these kernels are briefly introduced.

The Finite-Difference Time-Domain (FDTD) method [1] has been used as the standard numerical method for solving the Maxwell equations for over a half century. Its 4th-order version is known as “FDTD(2,4)” [2]. Although a higher-order finite difference reduces a numerical error in phase velocity, the Courant condition is more restricted. Recently a new scheme is developed for relaxing the Courant condition of FDTD(2,4) [3]. The implementation of the new FDTD method into a particle-in-cell simulation is discussed in the present study.

Numerical methods for solving the relativistic motion of charged particles with a higher accuracy is an issue for computational physics in various fields. The classic fourth-order Runge-Kutta method (RK4) [4] has been used over many years for tracking charged particle motions, although RK4 does not satisfy any conservation law. However, the Boris method [5] has been used over a half century in particle-in-cell plasma simulations because of its property of the energy conservation during the gyro motion. Recently, a new method for solving relativistic charged particle motions has been developed [6], which conserves both boosted Lorentz factor and kinetic energy during the gyro motion. However, the new integrator has the second-order accuracy in time and is less accurate than RK4. In the present study, a new fourth-order method has been developed by combining the new integrator [6] and RK4. The proposed method conserves boosted Lorentz factor and kinetic energy as

well during the gyro motion, but has higher accuracy than RK4.

It has been known that current densities without the continuity equation for charge includes a numerical error in electrostatic fields, which needs to be corrected by solving the Poisson equation implicitly. The charge conservation scheme for computing current densities has been discussed by many authors since early 1990's [7-11], which provides current densities that exactly satisfies the continuity equation for charge. Extension of the charge conservation scheme to higher-order particle shape function is discussed in the present study.

References

- [1] K. S. Yee, *IEEE Trans. Antennas Propag.*, **14**, 302—307 (1966)
- [2] P. G. Petropoulos, *IEEE Trans. Antennas Propag.*, **42**, 859—862 (1994)
- [3] H. Sekido and T. Umeda, *IEEE Trans. Antennas Propag.*, **71**, 1630—1639 (2023)
- [4] J. C. Butcher, *Appl. Num. Math.*, **20**, 247—260 (1996)
- [5] J. P. Boris, in *Proceedings of 4th Conference on Numerical Simulation of Plasmas*, 3—67 (1970)
- [6] T. Umeda, *J. Comput. Phys.*, **472**, 1111694 (2023)
- [7] J. W. Eastwood, *Comput. Phys. Commun.*, **64**, 252—266 (1991)
- [8] J. Villasenor and O. Buneman, *Comput. Phys. Commun.*, **69**, 306—316 (1992)
- [9] J. W. Eastwood, W. Arter, N. J. Brealey, and R. W. Hockney, *Comput. Phys. Commun.*, **87**, 155—178 (1995)
- [10] T. Esirkepov, *Comput. Phys. Commun.*, **135**, 144—153 (2001)
- [11] T. Umeda, Y. Omura, T. Tominaga, and H. Matsumoto, *Comput. Phys. Commun.*, **156**, 73—85, (2003)