

Analysis of the Distribution Function in the Landau Damping Process

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It is an interesting and useful topic to construct a fluid model for describing kinetic effects in collisionless plasmas, where particle distribution functions largely deviate from the Maxwellian distribution. One of the most famous researches on such a fluid model is the development of closures for fluid moment equations which can reproduce Landau damping[1].

As another related study, an analysis using a water-bag model has been proposed to approximately represent kinetic phenomena [2]. In the water-bag model, the one-dimensional distribution function $f(x, v, t)$ is approximated to a step-function divided by $N + 1$ velocity contours $V_j(x, t)$ ($N \in \mathbb{Z}$, $j = 1 \dots N + 1$) (Fig.1). Then, multiple systems with different degrees of freedom can be constructed hierarchically according to the value of the parameter N corresponding to the degrees of freedom. The water-bag model has model-dependent invariants called Casimir invariants. Using these Casimir invariants, we studied the kinetic effects of one-dimensional Vlasov-Poisson systems[2]. In our research, we succeeded in showing the relationship between the system of the water-bag model with different degrees of freedom and the Casimir invariants.

On the other hand, this result is limited to the case where the velocity contours $V_j(x, t)$ that constitute the water-bag model are single-valued functions. This condition turns out to be inapplicable to such cases as in Landau damping, for which $V_j(x, t)$ are multivalued. Numerical simulation based on the contour dynamics [3] is a powerful means to investigate how the velocity contours $V_j(x, t)$ become multivalued functions. The contour dynamics simulation is performed to clarify the structure of the distribution function $f(x, v, t)$ in the Landau damping process, and the simulation results are compared with the linear analytical solutions. In this comparison, we confirm that there are damped oscillations corresponding to complex frequency eigenvalues $\omega = \omega_r + i\gamma$ obtained by the kinetic dispersion relation [Fig. 2] in addition to the well-known oscillations due to the ballistic mode. Then, using the fact that the contour dynamics simulation obeys nonlinear Hamiltonian dynamics, we also examine the variation of the distribution function due to the quasilinear effect.

References

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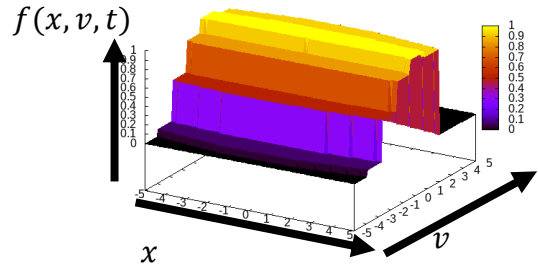


Fig.1 Schematic diagram of the distribution function $f(x, v, t)$ in the water-bag model

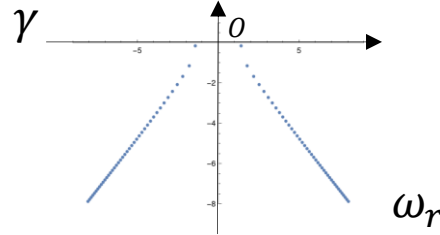


Fig. 2 Distribution of the complex frequency eigenvalues $\omega = \omega_r + i\gamma$ in the complex plane obtained by the kinetic dispersion relation.

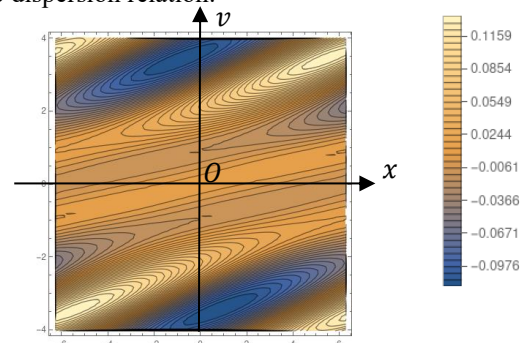


Fig. 3 An example of contours of $f_1(x, v, t)/f_{0M}(v)$ obtained by the contour dynamics simulation of the Landau damping where the distribution function is given by the sum of the background Maxwellian and perturbation parts as $f(x, v, t) = f_{0M}(v) + f_1(x, v, t)$.