Local Momentum Balance in Electromagnetic Gyrokinetic Turbulence

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Gyrokinetic theory and simulation provide a powerful framework to study microinstabilities and turbulent processes in magnetized plasmas. The gyrokinetic equations derived from the Lagrangian and/or Hamiltonian [1,2] describing the gyrocenter motion possess conservation properties, which are suitable for long-time and global transport simulations. Variational formulations based on the action integral of the Lagrangian provide a useful and systematic means to obtain governing equations and conservation laws of energy and momentum for considered systems. Especially, the momentum balance equation satisfied by the gyrokinetic model attracts our attention because it determines plasma flow profiles, which are considered as one of the key factors for improving plasma confinement.

In Ref. [3], the Eulerian formulation, which is also called the Euler–Poincaré reduction procedure, is applied to derive the governing equations of the Vlasov–Poisson–Ampère (or Vlasov–Darwin) system and those of the drift kinetic system in the general spatial coordinates, and the momentum balance equations for these systems are obtained by using the invariance of the action integrals for the systems under arbitrary spatial coordinate transformations. The resultant momentum balance equations contains the symmetric pressure tensor, which is derived by taking the variational derivative of the Lagrangian with respect to the metric tensor in the manner similar to that of the general theory of relativity. This derivation is in contrast to the conventional method using the spatial translation in which the asymmetric canonical pressure tensor generally enters the momentum balance equation.

In Ref. [4], the work in Ref. [3] is extended and the Eulerian variational formulation of the gyrokinetic system with electrostatic turbulence in general spatial coordinates is presented to derive the local momentum balance equation with the symmetric pressure tensor including the effects of electrostatic turbulent fluctuations on the momentum transport.

In the present study, the Eulerian formulation using the general spatial coordinates is applied to derivation of the local momentum balance equation in the gyrokinetic system with electromagnetic turbulence. The Lagrangian describing the electromagnetic gyrokinetic turbulence is presented in Ref. [5], where a novel second-order term is included to correctly derive both equilibrium and perturbation parts of the gyrokinetic Ampère's law. To derive the governing equations for the gyrocenter distribution functions and the turbulent electromagnetic fields, the Lagrangian is written as $L_{GKF} = L_{GK} + L_F$ where L_{GK} and L_F represent the gyrokinetic part (including the gyrocenter distribution functions) and the electromagnetic field part, respectively. The symmetric pressure tensor is given by

$$\Theta^{ij} = 2\frac{\delta L_{GKF}}{\delta g_{ij}}$$

where $\delta/\delta g_{ij}$ represents the variational derivative with respect to the metric tensor g_{ij}. Here, Θ^{ij} contains the pressure tensor due to particles' motion and the Maxwell electromagnetic tensor. Using the WKB representation, the turbulent part of Θ^{ij} is expressed in the form comparable with the one obtained in the past work of the momentum transport. In addition, effects of collisions on the momentum balance is considered. It is also shown that the Chew–Goldberger–Low (CGL) pressure tensor, which causes the neoclassical transport, is also included in Θ^{ij}.

References

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