Change of relaxation path by inclusion of Hamiltonian dynamics to simulated annealing of reduced magnetohydrodynamics

M. Furukawa[1] and P. J. Morrison[2]
[1]Faculty of Engineering, Tottori University, Japan,
[2]Department of Physics and Institute for Fusion Studies, University of Texas at Austin, Texas, USA
e-mail (speaker): furukawa@tottori-u.ac.jp

In simulated annealing (SA), we solve an initial-value problem of artificial dynamics derived from an original Hamiltonian system so that the energy of the system monotonically changes while the Casimir invariants are preserved. The energy minimum (extremum) is an equilibrium. We have applied the double bracket formulation of SA[1, 2] to reduced magnetohydrodynamics (MHD) for equilibrium calculations as well as stability analysis. In [3], we have developed a method to accelerate the relaxation of SA since equilibrium and stability analyses using relaxation are time consuming. In this study, we examine whether inclusion of the original Hamiltonian dynamics can accelerate the relaxation compared to that by SA only.

The original Hamiltonian dynamics adopted here is the low-beta reduced MHD[4]. The evolution equations for the vorticity U and the magnetic flux function ψ in cylindrical geometry are, after normalization,

$$\frac{\partial U}{\partial t} = [U,\varphi] + [\psi, J] - \varepsilon\frac{\partial J}{\partial \zeta} = f^1, \quad (1)$$

$$\frac{\partial \psi}{\partial t} = [\psi,\varphi] - \varepsilon\frac{\partial \varphi}{\partial \zeta} = f^2, \quad (2)$$

where φ is the stream function and J is the current density, ε is the inverse aspect ratio, $[f, g]$ is the Poisson bracket, and ζ is the toroidal angle coordinate.

The evolution equations of SA adopted here are

$$\frac{\partial U}{\partial t} = [U,\tilde{\varphi}] + [\psi, \tilde{J}] - \varepsilon\frac{\partial \tilde{J}}{\partial \zeta} = \tilde{f}^1, \quad (3)$$

$$\frac{\partial \psi}{\partial t} = [\psi,\tilde{\varphi}] - \varepsilon\frac{\partial \tilde{\varphi}}{\partial \zeta} = \tilde{f}^2, \quad (4)$$

where the artificial advection fields are defined by

$$\tilde{\varphi}(\boldsymbol{x},t) := -\alpha_{11}(t)\int_{\mathcal{D}} \mathrm{d}^3x'\, g(\boldsymbol{x},\boldsymbol{x}')f^1(\boldsymbol{x}',t), \quad (5)$$

$$\tilde{J}(\boldsymbol{x},t) := -\alpha_{22}(t)\int_{\mathcal{D}} \mathrm{d}^3x'\, g(\boldsymbol{x},\boldsymbol{x}')f^2(\boldsymbol{x}',t). \quad (6)$$

Here, g is a Green's function in three dimensions. The coefficients α_{11} and α_{22} may depend on time[3], although they should be both positive for monotonic decrease of the energy. Equations (3) and (4) have the same form as Eqs. (1) and (2), but the advection fields φ and J are replaced by the artificial ones $\tilde{\varphi}$ and $\tilde{J}$.

In this study, we solved

$$\frac{\partial U}{\partial t} = \tilde{f}^1 + cf^1, \quad (7)$$

$$\frac{\partial \psi}{\partial t} = \tilde{f}^2 + cf^2, \quad (8)$$

where c is a parameter expressing how much the original dynamics is included.

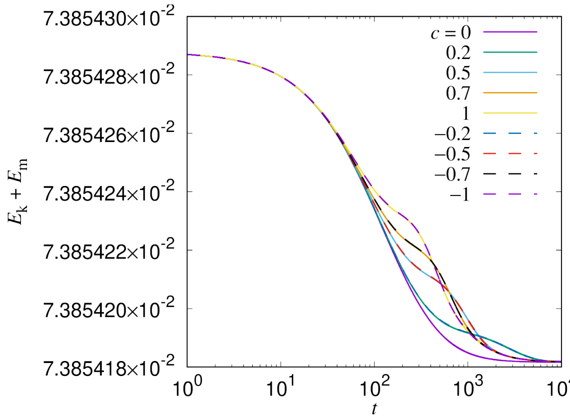

Figure 1: Time evolutions of total energy for various values of parameter c expressing the ratio of the original Hamiltonian dynamics to the SA. Inclusion of the original dynamics decelerated for the tested cases.

Figure 1 shows an example of time evolutions of total energy by Eqs. (7) and (8). The initial condition was taken to be a cylindrically symmetric equilibrium plus small-amplitude helical perturbations. The cylindrically symmetric equilibrium has no flow and is spectrally stable. The helical perturbations were taken to be dynamically accessible. The coefficients α_{11} and α_{22} were fixed as 100. The relaxation of energy was decelerated for the tested cases with either $c > 0$ or $c < 0$.

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