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Nonlinear Dispersion Relation for Dust Acoustic Wave based on Korteweg-de Vries model

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The dust acoustic wave (DAW) is supported in the plasma due to the dust inertia and plasma pressure balance. In dusty plasma experiments, these acoustic waves can excite spontaneously or by external perturbation [1] and may easily attain nonlinear characteristics [2]. In most such studies, linear theoretical models are proposed to explain the experimental dispersion of DAWs missing the impact of plasma nonlinearity. This paper proposes a theoretical model for nonlinear dispersion relation (NLDR) of DAWs. The model has been developed in the weak nonlinear dynamical regime of dust fluid and is based upon the Korteweg de-Vries (KdV) equation.

The proposed nonlinear dispersion relation, obtained from the exact solution of the KdV equation in which nonlinearity is quantified by the nonlinearity factor (κ), is given by:

$$\omega = \frac{4\,(K(\kappa))^2}{\pi^2}\left(\kappa^2 + \kappa - 1\right)\beta k^3$$

Here k and ω represent the wavevector and frequency, respectively. β represents the dispersion of the medium and can be written in terms of plasma parameters. The value of κ ranges between 0 and 1. At κ = 0, the equation represents purely linear dispersion relation (LDR), and at κ = 1, it has a nonlinear continuum dispersion representing a single soliton solution of the KdV equation. To check the effect of nonlinearity, we first spatio-temporarily evolved the solution of the KdV [3] and then took its Fourier transform (FT). The result is shown in Fig. 1, with bright spots. The first spot represents the fundamental mode, while the other spots represent the harmonics resulting from nonlinearity. Further, using the KdV-based NLDR for two different values of κ (0 and 0.98), we found that the nonlinearity leads to a shift in the frequency towards higher values. This result is shown in Fig. 1, represented by the yellow dash-dash (κ = 0) and red square (κ = 0.98) lines. We further compared the NLDR with the experimental result [1] in the long wavelength limit $k\lambda_D << 1$. We also plot the NLDR for three different values of κ with dash-dotted (κ = 0), dash-dash (κ = 0.7), and dotted (κ = 0.9) lines shown in Fig. 2, indicating the frequency shifting toward higher values.

We conclude that nonlinearity causes a frequency shift toward positive values. Therefore, there is a need to include the nonlinearity factor in the LDR of DAWs to properly explain the experimental dispersion relations.

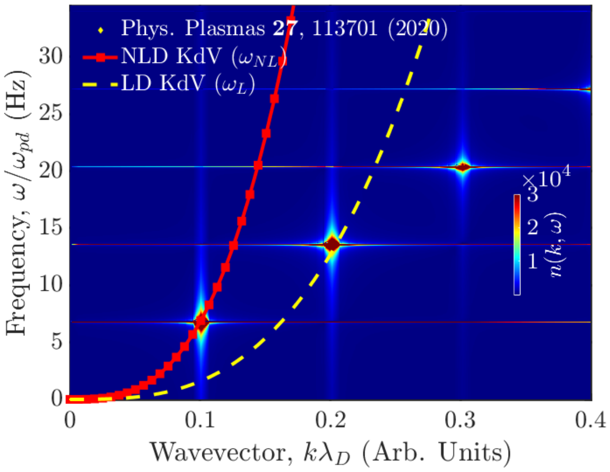

Fig 1. Comparison of NLDR (red square) and LDR (yellow dash-dash). The bright spots are obtained by taking the Fourier transform of the KdV solution.

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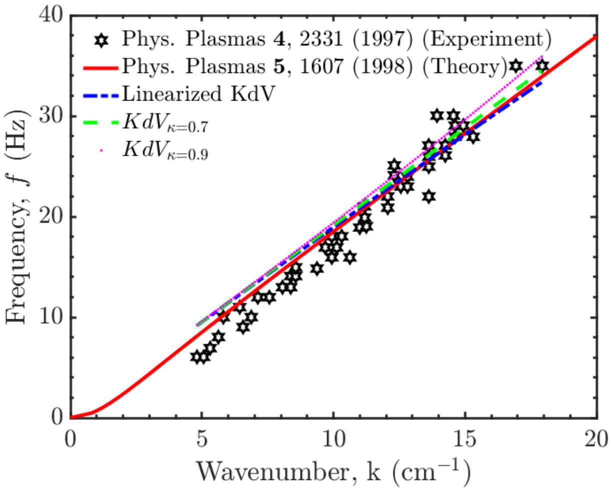

Fig 2. Comparison of the experimental DAW (black hexagon) [1] with LDR and the KdV NLDRs.