An extended form of the gyro-averaging operator in the presence of finite magnetic field variation

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In the description of magnetized plasma dynamics under the influence of electromagnetic fluctuations, it is convenient to study motion of the gyrating center (the gyrocenter) instead of full particle motion. The fast gyromotion around the magnetic field is averaged out in this approach. Then, a gyrocenter experiences averaged electromagnetic fluctuations during its gyration. This gyro-averaged field or potential is evaluated by a gyro-averaging operation of the physical quantity under consideration. In this contribution, we derive expressions for the gyro-averaging operator which are applicable to electrostatic fluctuations in the presence of finite inhomogeneity in the confining magnetic field. A mathematical analysis is carried out of the gyro-average operation given by the generalized expression[1]

$$e^{-iy_c X}\left[\frac{1}{2\pi}\int_0^{2\pi} e^{-iX(\cos\alpha - y_c\cos 2\alpha + y_s \sin 2\alpha)}d\alpha\right]$$

where $X=k_\perp\rho_0$ with ρ_0 the local Larmor radius, $y_c=(\rho_0/4)\nabla_\perp|\ln B|\cos\gamma$, $y_s=(\rho_0/4)\nabla_\perp|\ln B|\sin\gamma$ with γ the angle between $\mathbf{k}_\perp$ and $\nabla_\perp \ln B$, and α is the gyro-angle. See figure 1 for geometry corresponding to particle gyration in this study. These expressions are expected to provide more accurate computation of density fluctuations, hence the potential being used in GK Poisson equation.

In the low wavenumber limit, the generalized gyro-averaging operator is shown to be represented by sums of Bessel functions with different orders. A simplified expression is provided as a Pade approximant in the low wavenumber limit,

$$\begin{aligned}\mathcal{G} &\simeq \frac{J_0(X)J_0(X)}{J_0(X)-\mathcal{F}(k_\perp)\mathcal{T}(k_\perp)} \\ &= \frac{e^{-b}}{e^{-b/2}\left[1-e^{b/2}\mathcal{F}(k_\perp)\mathcal{T}(k_\perp)\right]} \\ &= \frac{e^{-b/2}}{1-e^{b/2}\mathcal{F}(k_\perp)\mathcal{T}(k_\perp)},\end{aligned}$$

where $b = X^2 = k_\perp^2\rho_0^2, J_0(X)\simeq e^{-b/2}$ and

$$\mathcal{F}(k_\perp) = \left(\frac{\rho_0}{2}\right)^2 \mathcal{D}(k_\perp),$$

$$\mathcal{T}(k_\perp) = 1+\left(\frac{k_\perp\rho_0}{2}\right)^2 - \frac{1}{3}\left(\frac{k_\perp\rho_0}{2}\right)^4 = 1+\frac{b}{4}-\frac{b^2}{48}.$$

This form is a generalization of the conventional operator that has been developed in Ref. [2]. It could be used in practical computations based on the gyrofluid formulation.

In the high wavenumber limit, we make use of the method of stationary phase to compute the gyro-averaging operator. In this asymptotic calculation, we find that the operator reduces to the form,

$$\mathcal{G} = \sqrt{\frac{2}{\pi}}e^{i\pi/4}(\rho_0\nabla_\perp)^{-1/2}\left[\cos(X-\pi/4)+i2y_c\sin(X-\pi/4)\right].$$

This high wavenumber limit form naturally involves in the fractional derivative, which contains nonlocality automatically. In particular, we emphasize that this result has been derived without physical assumptions that are often made to set up a transport equation in terms of a fractional derivative. A further discussion on a potential impact of this asymptotic expression in the high wavenumber limit, in particular in relation to the impact of energetic particles on micro-turbulence, will be made in the conference.

References

[1] S. S. Kim and Hogun Jhang, Phys. Plasmas, **27**, 092305 (2020)
[2] W. Dorland and G. W. Hammett, Phys. Fluids B 5, 812 (1993)

Figure 1. Geometry describing a particle gyration in the presence of finite magnetic field inhomogeneity within a Larmor radius.