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On gyroaveraging in drift-kinetic simulations of fast particle dynamics

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Gyroaveraging in numerical simulations is often necessary for obtaining a realistic spectrum of resonant instabilities and realistic saturation amplitudes because the effective strength of wave-particle interactions in a magnetized plasma depends on the ratio $\rho / \lambda_\perp$ of gyroradius and perpendicular wavelength. A rigorous recipe is provided by gyrokinetic theory, which splits the background field $\boldsymbol{B}_{\rm ref}$ from the fluctuations $\delta\boldsymbol{B}$. Only the latter are gyroaveraged and the motion of charged particles is then represented by the motion of gyrocenters. Here, we are interested in formulations that make use of physical electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$ (rather than their potentials Φ and $\boldsymbol{A}$), which requires that Faraday's law $\partial\boldsymbol{B}/\partial t = -\nabla\times\boldsymbol{E}$ and the solenoidal condition $\nabla\cdot\boldsymbol{B} = 0$ are both enforced explicitly. In this case, rigorous gyroaveraging in a gyrokinetic model requires that $\boldsymbol{E}$ and $\delta\boldsymbol{B}$ are subject to two-dimensional gyrodisk integrals (Stokes' theorem), which is computationally expensive.

In practice, it can be meaningful to sacrifice some rigor for convenience and computational speed, if the errors made are tolerable. For instance, the gyrodisk integral in a gyrokinetic model formulated in terms of $\boldsymbol{E}$ and $\boldsymbol{B}$ may be approximated by a reduced one-dimensional gyrocircle integral, although this is justified only when the gyroradius is guaranteed to be small compared to the perpendicular wavelength for all simulation particles [1].

Here, we consider another choice that is often used in kinetic orbit-following codes and MHD-PIC hybrid simulations of fast particles: instead of gyrokinetics, one solves drift-kinetic equations of motion while performing gyrocircle-averaging in the poloidal (R, z) plane around guiding centers. Hamiltonian guiding center theory does not incorporate gyroaveraging, and the impact of *ad hoc* gyroaveraging on conservation laws was recently analyzed [2]. The example studied was a JT-60U plasma containing beam ions (deuterons) with a kinetic energy of $Mv_0^2/2$ = 400 keV and fairly large gyroradius: $\rho_0 = Mv_0 / eB_0 \sim 10$ cm.

Conventional gyroaveraging using $\rho(B) = Mv_\perp(B) / eB_{\rm ref}$, with $B = |\boldsymbol{B}|$ yields fields $\langle\boldsymbol{E}\rangle$ and $\langle\boldsymbol{B}\rangle$ that violate Faraday's law: $\partial\langle\boldsymbol{B}\rangle/\partial t \neq \langle\partial\boldsymbol{B}/\partial t\rangle = -\langle\nabla\times\boldsymbol{E}\rangle \neq -\nabla\times\langle\boldsymbol{E}\rangle$. However, we found that the errors in the form of spurious heating were tolerable or hardly noticeable at all as in the example shown in Fig.1(c,d). For comparison, Fig.1 also shows results for (a,b) Hamiltonian guiding center motion, and (e,f) conservative gyroaveraging [2], where $\langle\delta\boldsymbol{B}\rangle$ is replaced by a modified field satisfying $\partial_t\delta\boldsymbol{B}_{\rm mod} = -\nabla\times\langle\boldsymbol{E}\rangle$ (a procedure that one may refer to as "Faraday cleaning").

Gyroaveraging in a guiding center simulation can be viewed as a physically motivated smoothing operation on the fields. Different choices will have different effects on a simulation's conservation properties. Without in-depth analysis, the outcome may even seem counter-intuitive at first glance as in Fig.1(g,h), which shows the results of a simulation where we used a constant gyroradius ρ_0 instead of $\rho(B)$ with B taken at the current guiding center position. In this case, one may expect to be making only a small error $|\langle\nabla\times\boldsymbol{E}\rangle - \nabla\times\langle\boldsymbol{E}\rangle| \sim |\boldsymbol{E}(\boldsymbol{x}+\Delta\boldsymbol{x}) - \boldsymbol{E}(\boldsymbol{x})| / \rho_0$, but Fig.1(g,h) shows that this error tends to accumulate here near the resonance, yielding noticeable spurious heating.

We also found that, in the presence of resonant fluctuations that resemble "normal" (eigen)modes of the system, spurious heating tends to be slow and becomes significant only on a time scale of 10 ms, where collisions and sources must also be considered. In contrast, spurious heating was found to be much faster in the presence of strongly distorted "nonnormal" fluctuation patterns [2]. While this amplification effect remains to be understood, it can be useful for code verification (as done in Fig.1). Of course, in reality, such nonnormal patterns tend to be short-lived and have rapidly varying phases, which should reduce their net spurious heating effect.

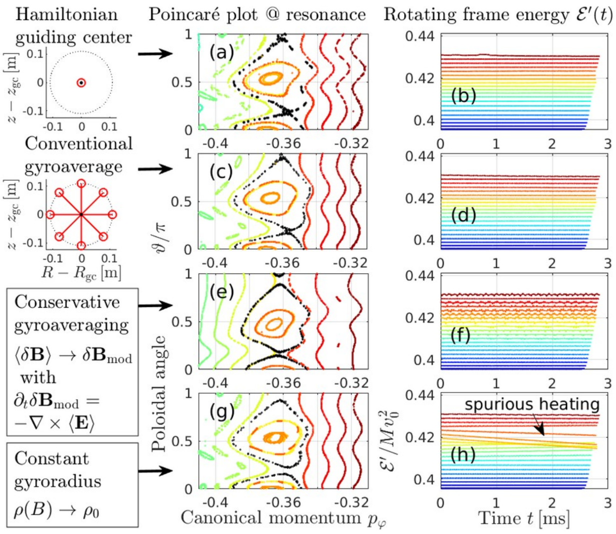

Figure 1. Conservation properties of a guiding center simulation without (a,b) and with gyroaveraging (c-h). The central column shows Poincaré plots of a portion of a resonance. The right column shows time traces of the rotating frame energy $\mathcal{E}' = E + \omega P_\varphi / n$, which should be conserved in the present setup. See Ref. [2] for details.

References

[1] Porazik & Lin, *Commun. Comput. Phys.* **10** (2011) 899.

[2] Bierwage & Shinohara, *Phys. Plasmas* **29** (2022) 113906.

Fig.1(c,d) above is a corrected version of Fig.14(w,x) of [2], where a coding error caused enhanced spurious heating.