Integro-differential analysis of ion cyclotron waves in tokamak plasmas

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Propagation and absorption of waves in hot plasmas is generally described by an integro-differential equation with the integral form of dielectric tensor. The integral along the particle orbit is a natural way to represent the response of hot plasma to the wave field. In most of previous wave analyses, however, the integral form is approximated by means of wave number in cold plasmas, differential form, or Fourier-transform. Since such essentially local approximation cannot describe short-wavelength waves, cyclotron damping in an inhomogeneous magnetic field, and stochastic heating near the reflection point of the waves, a systematic scheme for integro-differential full wave analysis has been developed.

Maxwell's equation with the integral form of dielectric tensor is written as

$$\nabla\times\nabla\times\boldsymbol{E}(\boldsymbol{r})+\frac{\omega^2}{c^2}\int\overleftrightarrow{\epsilon}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{E}(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}=\mu_0\boldsymbol{J}_{ext}(\boldsymbol{r})$$

The response of the particle motion along the field line is described by the plasma dispersion kernel function (PDKF),

$$U_n(\xi,\eta)=\frac{\mathrm{i}}{\sqrt{2\pi}}\int_0^\infty \mathrm{d}\tau\,\tau^{n-1}\,\exp\left[-\frac{1}{2}\frac{\xi^2}{\tau^2}-\frac{1}{2}\eta^2\tau^2+\mathrm{i}\,\tau\right]$$

where ξ denotes the distance z-z' divided by the particle excursion length v_T/ω and η the effect of inhomogeneity. In a uniform plasma, PDKF is the Fourier inverse transform of the plasma dispersion function. When the strength of the magnetic field has an extremum along the field line, the second-order PDKF has to be introduced. The response of the perpendicular motion is described by the plasma gyro kernel functions (PGKF),

$$F_n^{(i)}(X,Y) \equiv \frac{1}{2\pi^2}\int_0^\pi \mathrm{d}\theta\exp\left[-\frac{X^2}{1+\cos\theta}-\frac{Y^2}{1-\cos\theta}\right]f_n^{(i)}(\theta)$$

where X denotes the distance between the middle point $(x+x')/2$ and the gyro center and Y the distance x-x', both normalized by the gyro radius, and $f_n^{(i)}$ denotes the trigonometric functions of θ and $n\theta$. They are related to the Fourier inverse transform of the modified Bessel function multiplied by the Gaussian function [1].

For the numerical analysis of integro-differential equation, the finite-element-method (FEM) in which the coupling within one element is usually considered is extended to include the coupling between the elements within the particle excursion length.

One-dimensional integro-differential analyses has been made in various inhomogeneous plasmas, such as the laser-plasma resonant interaction in a plasma sheath, the cyclotron damping in a magnetic beach, and the O-X-B mode conversion of electron cyclotron waves[2].

The two-dimensional integro-differential equation solver using FEM has been newly developed and applied to the analysis of ICRF wave propagation and absorption in tokamak configuration. The wave structure in the poloidal cross section is compared with the results of cold plasma approximation and the Fourier mode expansion with warm plasma approximation. The effects of short-wavelength modes (ion Bernstein waves, electrostatic ion cyclotron waves) and inhomogeneity of the magnetic field on the ion cyclotron damping are discussed.

References

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