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Vorticity wave interaction and exceptional points in shear flow instabilities

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Interaction of vorticity waves demonstrates the mechanism of shear flow instabilities [1], and naturally provides a non-modal approach to study optimal growth and transient dynamics in such instabilities [2]. In the dynamical system of counter-propagating vorticity waves [3], the onset of shear instabilities corresponds to a bifurcation of fixed points from two neutral centers to a pair of stable and unstable nodes.

On the other hand, the role of symmetries and symmetry-breaking bifurcations in fluid dynamics is long recognized [4], and it is recently shown via *spectral analysis* that shear flow instabilities are the result of spontaneous PT-symmetry breaking [5]. The results, however, have not yet been related to the *dynamics* of vorticity waves.

In this work, we establish a link between vorticity wave interaction and PT-symmetry breaking in shear flow instabilities. We show that the dynamical system of counter-propagating vorticity waves is a non-Hermitian system with saddle-node exceptional points. Vorticity wave dynamics are directly linked to the PT-symmetry-breaking bifurcation of fixed points (Fig.1) and the Krein collision between eigenmodes with opposite Krein signatures. The key parameter to control the bifurcation is the ratio between frequency detuning and coupling strength of the vorticity waves.

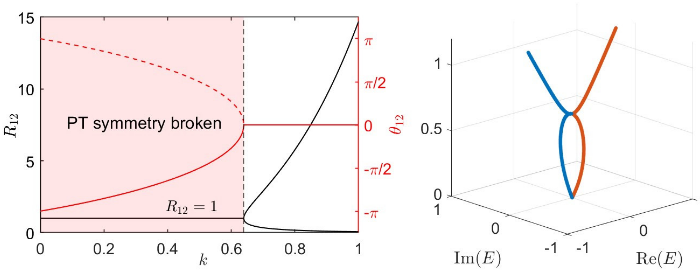

Fig. 1. (Left) Parameter space of two coupled vorticity waves and bifurcation of fixed points with amplitude ratio $R_{12} = Q_1/Q_2$ and relative phase $\theta_{12} = \theta_1 - \theta_2$. When the control parameter $k \to k_c = 0.64$, the system exhibits a PT-symmetry-breaking bifurcation and the two vorticity waves become phase-locked. (Right) The eigenspectra $E_{1,2}(k)$. The saddle-node exceptional point locates at $k = k_c$ in the complex $E - k$ space.

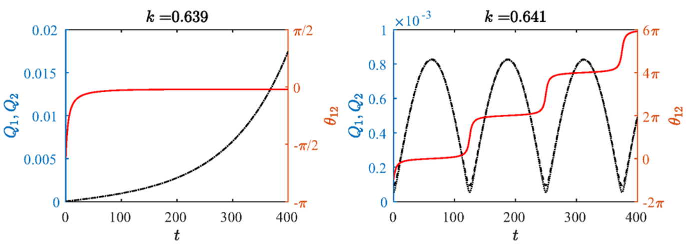

Fig. 2. Amplitude and phase dynamics of two coupled vorticity waves, from a phase-slip state $(k = 0.641 \sim k_c^+)$ to a phase-locked state $(k = 0.639 \sim k_c^-)$.

The most striking feature of non-Hermitian systems is the existence of exceptional points [6]. We show that the transition of phase dynamics from a phase-slip state to a phase-locked state near the exceptional point corresponds to spontaneous PT-symmetry breaking and the onset of shear instabilities. (Fig.2) The phase-locked state leads to exponential growth of the shear instabilities, while the phase-slip dynamics lead to non-modal, transient growth of initial perturbations. Near the exceptional point, the phase-slip frequency $\Omega \propto |k - k_c|^{1/2}$ shares the same critical exponent 1/2 with the phase rigidity of system eigenvectors.

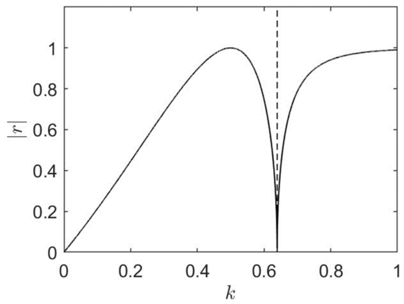

Fig. 3. Critical behavior of the system around the exceptional point, $|r| \propto |k - k_c|^{1/2}$. The phase rigidity $|r|$ measures biorthogonality of eigenvectors.

We then extend the analysis to interaction of multiple vorticity waves [7]. Multiple exceptional points exist in the system, as shown in Fig. 4.

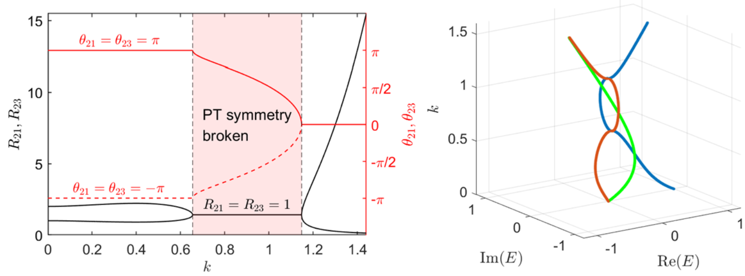

Fig. 4. (Left) Parameter space of the three coupled vorticity wave system. When the control parameter $k_{c1} < k < k_{c2}$, PT-symmetry is spontaneously broken and the three vorticity waves become phase-locked. (Right) The eigenspectra $E_{1,2,3}(k)$ in the complex $E - k$ space. The saddle-node exceptional points locate at $k = k_{c1}$ and $k = k_{c2}$.

In general, it is shown that the framework of vorticity wave interaction is naturally related to non-Hermitian physics and PT-symmetry-breaking bifurcations. Therefore, we anticipate further applications of this framework to the analysis of various magnetohydrodynamic instabilities in plasmas [8].

References

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