High-order field theory and weak Euler-Lagrange-Barut equation for classical relativistic particle-field systems

Peifeng Fan[1], Qiang Chen[2], Jianyuan Xiao[3], Zhi Yu[4]

[1] School of Physics and Optoelectronic Engineering, Anhui University, [2] National Supercomputing Center in Zhengzhou, Zhengzhou University, [3] School of Nuclear Science and Technology, University of Science and Technology of China, [4] Institute of Plasma Physics, Chinese Academy of Sciences

e-mail (speaker): pffan@ahu.edu.cn

Abstract: In both quantum and classical field systems, conservation laws such as the conservation of energy and momentum are widely regarded as fundamental properties. A broadly accepted approach to deriving conservation laws is built using Noether's method. However, this procedure is still unclear for relativistic particle-field systems where particles are regarded as classical world lines.

In the present study, we establish a general manifestly covariant or geometric field theory for classical relativistic particle-field systems. In contrast to quantum systems, where particles are viewed as quantum fields, classical relativistic particle-field systems present specific challenges. These challenges arise from two sides. The first one comes from the mass-shell constraint. To deal with the constraint of mass-shell, the Euler-Lagrange-Barut (ELB) equation is used to determine the particle's world lines in the 4D Minkowski space. Besides, the infinitesimal criterion which is a differential equation in the formal field theory, is reconstructed by an integro-differential form. The other difficulty is that fields and particles depend on heterogeneous manifolds.

To overcome this challenge, we propose using a weak version of the ELB equation that allows us to connect local conservation laws and continuous symmetries in classical relativistic particle-field systems. By applying the weak ELB equation to classical relativistic particle-field systems, we can systematically derive local conservation laws by examining the underlying symmetries of the system. Our proposed approach provides a new perspective on understanding conservation laws in classical relativistic particle-field systems.

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