



## Hamiltonian formulations of quasilinear theory for magnetized plasmas

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The complex interaction between charged particles and electromagnetic-field wave fluctuations in a magnetized plasma represents a formidable problem with crucial implications toward our understanding of magnetic confinement in laboratory and space plasmas [1]. These wave-particle interactions can be described either linearly, quasi-linearly, or nonlinearly, depending on how the background plasma is affected by the fluctuating wave fields and the level of plasma turbulence associated with them [2].

In linear plasma wave theory [3], where the field fluctuations are arbitrarily small, the linearized perturbed Vlasov distribution of each charged-particle species describes the charged-particle response to the presence of small-amplitude electromagnetic waves which, when coupled to the linearized Maxwell wave equations, yields a wave spectrum that is supported by the uniform background magnetized plasma [3].

In weak plasma turbulence theory [4], the background plasma is considered weakly unstable so that a (possibly discrete) spectrum of field perturbations grows to finite but small amplitudes. While these small-amplitude fluctuations interact weakly among themselves, they interact strongly with resonant particles, which satisfy a wave-particle resonance condition in particle phase space (described in terms of unperturbed particle orbits). These resonant wave-particle interactions, in turn, lead to a quasilinear modification of the background Vlasov distribution on a long-time scale compared to the fluctuation time scale [5,6].

Lastly, in strong plasma turbulence theory [7], nonlinear wave-wave and wave-particle-wave interactions cannot be neglected, and wave-particle resonances include perturbed particle orbits [4].

Our primary purpose is to present complementary views of two-dimensional quasilinear diffusion in a uniform magnetized plasma. First, we review the (Newtonian) quasilinear theory derived by Kennel and Engelmann [8] which represents the paradigm formulation upon which

many subsequent quasilinear formulations are derived [3].

As an alternative formulation of quasilinear theory, we present a Hamiltonian formulation that relies on the use of guiding-center theory for a uniform magnetic field. In this Hamiltonian formulation, the quasilinear diffusion equation is described in terms of a diffusion tensor whose structure is naturally generalized to three-dimensional quasilinear diffusion in a nonuniform magnetized plasma.

We conclude our presentation by discussing the works of Brizard and Chan [9-10], in which the Hamiltonian quasilinear diffusion of relativistic particles in the presence of electromagnetic fluctuations in an axisymmetric dipole geometry is presented.

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