Kinetic closure with a charged disk model

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Motivation The BBGKY hierarchy is the starting point for deriving the Vlasov equation with a collision operator. With a large plasma parameter g^{-1}one justifies eliminating 2-point correlations in terms of the 1-point function. Numerically testing the validity of these assumptions is prohibitive. because of high dimensionality, In this presentation, we will present a 1-D model composed of interacting aligned charged disks so as to address the validity of the Bogoliubov assumption in a computable model. A system of N classical particles obeys the Liouville equation for the distribution function (d.f.) in 6N-dimensional phase space

$$\frac{\partial F^{(N)}}{\partial t} + \sum_{i=1}^{N} \frac{\mathbf{p}_i}{m_i} \frac{\partial F^{(N)}}{\partial \mathbf{q}_i} + \sum_{i=1}^{N} \mathcal{F}_i \frac{\partial F^{(N)}}{\partial \mathbf{p}_i} = 0$$

with $\mathcal{F}_i$ the total force acting on the i^{th} particle. By integration over the variables of a subset of particles, the Liouville equation can be transformed into a chain of equations connecting the evolution of s-particle d.f. $F^{(s)}$ with the $(s+1)$-d.f. $F^{(s+1)}$. By truncating this chain by neglecting terms of order g^2 we obtain a closed system involving $F^{(1)}$ and $F^{(2)}$. Bogolyubov's approach postulates that $F^{(2)}$rapidly reaches an asymptotic form, uniquely determined by the instantaneous form of $F^{(1)}$.
In 3-D space the equation for the time evolution of $F^{(2)}$ is13-dimensional (one time plus the 12-dimensional two-particle phase space). In 1-D space the equation for $F^{(2)}$ (actually the two-foil d.f.) is 5-dimensional (time plus 4-dimensional phase space). However in 1-D the electric field generated by a charge foil does not decay with distance so that the two-foil interaction energy diverges at infinity. This makes it unclear how to define a finite correlation length. A possible compromise is to introduce an interaction potential that behaves as that of a 1-D foil at short distances and decays as the inverse of the distance at large distances. Any interaction potential of this type introduces by necessity a length scale not present in the Coulomb interaction. A possible choice [1] is to consider ta system of aligned charged disks where the characteristic length is the radius b of the disk. In this case the electrostatic interaction energy is finite both at zero and at infinite distance between disks.

Model The interaction energy between two uniformly charged thin disks of radius b and charge $Q_{1,2}$ is given by [2] $(4/b)Q_1Q_2V(\xi)$, with $\xi = x/b$t$|x_1 - x_2|/b$ and $V(\xi) = -(\xi/2)\{1 - 1/(3\pi)[(4-\xi^2)E(-4/\xi^2)$ $+(4+\xi^2)K(-4/\xi^2)]\}$, with E and K elliptic integrals and $V(\xi=0) = 4/(3\pi)$, $V(\xi) = 1/(4\xi)$ for $\xi \to \infty$.
The spatial spectrum of $V(\xi)$ differs from that of the Coulomb potential at large wavelengths $kb \ll 1$, where its F.T. $\hat{V}(kb) = (1/8)[Const - 2\ln[(kb)^2] - .]..$ while for $(kb)^2 \gg 1$, $\hat{V}(kb) = 1/(kb)^2$. The logarithmic term at small wave-numbers arises in 1-D from the $1/\xi$ dependence of the interaction potential at large distances.
Defining the densities $n_{i,e}(x,t)$ (disks per unit length) and the charge density $\rho(x) = Q\,[n_i - n_e(x,t)]$ and taking ion disks immobile, the cold fluid equations, $\partial_t n_e + \partial_x(n_e u_e) = 0$, $M_e n_e\left(\partial_t + u_e\partial_x\right)u_e = n_e\mathcal{F}_e$.

$$\mathcal{F}_e(x) = -\frac{4Q_e}{b}\frac{d}{dx}\int_{-\infty}^{+\infty} dx'\,\rho(x')V(|x-x'|/b)\,,$$

give the dispersion relation $\omega^2 = \omega_{Dpe}^2[(kb)^2\,\hat{V}(kb)]$ where $\omega_{Dpe}^2 = (4Q^2n_o)/(M_eb^2)$. For $kb \gg 1$ the disk-plasma waves become Langmuir waves with frequency ω_{Dpe}. For $kb \ll 1$ we find (logarithmically corrected) ``cold electron-disk sound'' waves of the form $\omega^2 = k^2[C_1 + C_2\ln(kb)^2]v_D^2$, where $C_{1,2}$ are constants and b$v_D^2 = b^2\omega_{Dpe}^2 = 4Q^2n_o/M_e$. The phase velocity of Langmuir waves is smaller than v_D while for old sound waves we have $\omega^2/k^2 \sim [\ln(kb)]^2\,v_D^2 >> v_D^2$.
The disk screened potential at thermodynamic equilibrium exhibits an exponential-type behaviour for small x/b while it decreases as $(x/b)^{-s}$with $s \gtrapprox 1$ for large x/b. This tail originates from the logarithmic contribution to $\hat{V}(kb)$ in the limit of small kb.
The derivation of the Vlasov equation follows standard lines with the particle density as the independent variable

$$\frac{\partial f_j(x,v,t)}{\partial t} + v\frac{\partial f_j(x,v,t)}{\partial x} + \frac{\mathcal{F}_j(x,t)}{M_j}\frac{\partial f_j(x,v,t)}{\partial v} = 0,$$

with $f_j(x,v,t)$ s the d.f. of the j species.

Present conclusions In order to model correlation effects in a 3-D plasma using a reduced dimensionality configuration, it may be convenient to define an effective interaction energy constructed so as to retain the physical effects that we deem must be included in order to obtain an informative model. A numerical project is under way in order to investigate the kinetic properties of this 1-D plasma model and to integrate numerically the equation for $F^{(2)}$ with given initial conditions.

References

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