Formulation of a self-consistent reduced transport theory for discrete plasma waves near their instability threshold

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A quasilinear (QL) plasma transport theory that incorporates Fokker-Planck dynamical friction (drag) and pitch angle scattering is self-consistently derived from first principles for an isolated, marginally-unstable mode resonating with an energetic minority species. It is found that drag fundamentally changes the structure of the wave-particle resonance, breaking its symmetry and leading to the shifting and splitting of resonance lines [1]. In contrast, scattering broadens the resonance in a symmetric fashion [2]. The work shows how to systematically calculate the resonance function, i.e. the envelope function that weights the strength of the resonant interaction, in QL theory in the presence of collisions. It is demonstrated that a QL system that employs the calculated broadening functions (Fig. 1) recently reported [1] automatically recovers the nonlinear growth rate and mode saturation levels for near-threshold plasma instabilities previously calculated from a significantly more complex nonlinear kinetic theory based on solving a time delayed integro-differential equation [3,4]. Moreover, it is shown that a QL theory can be constructed for a single resonance, provided that it experiences enough background stochasticity. The results enable realistic reduced modeling of diffusive transport observed in fusion devices.

Comparison with fully nonlinear simulations shows that the proposed quasilinear system preserves the exact instability saturation amplitude and the corresponding particle redistribution of the fully nonlinear theory. Even in situations in which drag leads to a relatively small resonance shift, it still underpins major changes in the redistribution of resonant particles. This novel influence of drag is of interest over a range of disciplines that employ kinetic theory, being equally important in plasmas and self-gravitating systems. In fusion plasmas, the effects are especially pronounced for fast-ion-driven instabilities in tokamaks with low aspect ratio or negative triangularity, as evidenced by past observations. The same theory directly maps to the resonant dynamics of the rotating galactic bar and massive bodies in its orbit, providing new techniques for analyzing the bar deceleration and the radial migration in galactic dynamics [5].

The present formulation gives simple analytical answers to the shape of discrete-resonance collisional functions in quasilinear theory, for both Fokker-Planck scattering and drag collisions. These functions, that integrate to unity, replace a simple delta function that appears in the diffusion coefficient for the case of no broadening. It is shown that the knowledge of these functions removes a major arbitrariness with respect to previous resonance broadening approaches, which consisted of tuning broadening parameters to match the expected saturation levels [6]. In addition, analytical forms for the modification of the distribution function can be naturally constructed. These forms are particularly useful for code verification. This formulation is required to model and verify the fast ion relaxation in tokamaks upon interaction with Alfvénic eigenmodes being implemented in the Resonance Broadened Quasilinear (RBQ) code [7], where particle diffusion occurs along the canonical toroidal momentum and the energy of the beam ion distribution function.
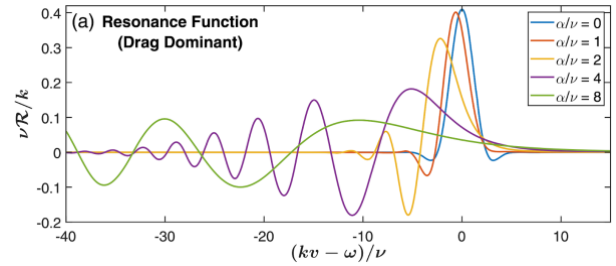

Figure 1: The resonance function is a measure of the strength of resonant wave-particle energy exchange. It is shown for several ratios of collisional drag frequency (α) over the diffusive scattering frequency (ν), illustrating how scattering broadens the resonances while drag shifts and splits them into multiple peaks.

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