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A Hamiltonian model and its structure preserving discretization of the ponderomotive force in 1D

William Barham[1,2], Yaman Güçlü[4], Philip J. Morrison[2,3], and Eric Sonnendrücker[4,5]
[1] Oden Institute for Computational Science Engineering and Sciences, University of Texas Austin
[2] Institute for Fusion Studies, University of Texas Austin
[3] Department of Physics, University of Texas Austin
[4] Max Planck Institute for Plasma Physics, Numerical Methods in Plasma Physics
[5] Department of Mathematics, Technical University of Munich
e-mail (speaker): william.barham@utexas.edu

A large number of interactions between matter and the electromagnetic field may be adequately captured by approximate models which reduce the interaction to a nonlinear polarization of the medium. In many ways, this concept is the heart of nonlinear optics, but also bears relevance in plasma physics in which asymptotic approximations often induce field dependent polarization and magnetization of the electromagnetic fields: see e.g. [1] for nonlinear electromagnetic media arising from lifting particle-orbit asymptotics to kinetic theories. In this work, we derive a general Hamiltonian structure preserving discretization of Maxwell's equations in nonlinear media which might be applied to broad class of models, derive a self-consistent Hamiltonian model of the ponderomotive force to which we might apply this general discretization procedure, and finally investigate the qualities of the general method as applied to this particular system.

A well known laser-plasma interaction model gives rise to a nonlinear polarization of the electromagnetic medium. In the presence of an inhomogeneous oscillatory electric field, charged particles experience a net force, averaged over the oscillatory timescale, known as the ponderomotive force. Starting from the two-fluid equations, we derive a Hamiltonian model in which the ponderomotive force provides a current source for Maxwell's equations. The asymptotic assumptions under which this model is derived correspond to what is frequently called the "linear regime" in laser-plasma interaction models [2]. Our model is possibly the simplest self-consistent model of the ponderomotive force, and closely resembles the model studied in [3]. Moreover, our model is perhaps the first to elucidate the Hamiltonian structure of the 1D linear regime, and provides an alternative approach to its derivation based on applying the asymptotics directly to the energy functional and Poisson bracket rather than the equations of motion. We call the system of equations so derived the 1D ponderomotive Maxwell system.

One finds a simple Poisson bracket for the 1D ponderomotive Maxwell system: it is a direct sum of the Poisson brackets for Maxwell and acoustic wave equations, as well as a coupling bracket. Because all of these brackets are field-free, a structure preserving discretization is easily accomplished using finite element exterior calculus (FEEC). In particular, we use a spectral element FEEC method. We explore the behavior of discretizations based on not only a traditional spectral element FEEC approach, but also a discontinuous broken-FEEC approach, see [4,5]. Temporal discretization is accomplished using Hamiltonian splitting.

Using both conforming and broken-FEEC methods with both low and high order discretizations in space and time, we simulate the 1D ponderomotive Maxwell system in order to better understand the properties of this new approach to discretizing Maxwell's equations in nonlinear media. In all cases, we find good conservation of energy to the order of the splitting method used, and conservation Casimir invariants (e.g. Gauss's laws) to machine precision. To our knowledge this is the first time-domain solver for Maxwell's equations in nonlinear media which conserves energy and Gauss's laws, accommodates high order discretizations in both space and time, and which may use local, discontinuous basis functions to localize nonlinear solves (allowing for parallel implementation). The methods used to discretize the 1D ponderomotive Maxwell system shows promise to generalize well to a broad class of models of electrodynamics in nonlinear media. Hence, these favorable qualities found in method used to study the 1D ponderomotive Maxwell system might be used to study a much broader class of models in nonlinear optics and plasma physics.

References

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