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Extended magnetohydrodynamic approach to plasma-vacuum interface

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The magnetohydrodynamic (MHD) equations are widely used as the macroscopic fluid model for plasma and have been solved analytically and numerically for various applications. The MHD equations are originally derived from the two-fluid equations (for ions and electrons) by assuming quasi-neutrality and neglecting the Hall and electron-inertia effects. However, this MHD approximation is invalid when plasma density decreases to a certain level. The plasma-vacuum interface (i.e., the free boundary of plasma) is therefore difficult to treat in the standard MHD. For example, the phase speed of Alfvén wave diverges unphysically in the low-density limit.

In fact, the Hall and electron-inertia effects are nonnegligible in the scales of ion and electron's inertial lengths, respectively, and they are inversely proportional to the square root of density. While the dense plasma region is well described by MHD, such the two-fluid effects should be taken into account in the region where the density decays to zero (i.e., vacuum). For this reason, we are naturally led to study the extended MHD (XMHD) model [1], in which the Hall and electron-inertia terms are restored while quasi-neutrality is still assumed (by taking the light speed infinite). The collisional effects such as resistivity and viscosity are also taken into account in this study. The XMHD equations are, then, well-posed in both plasma and vacuum regions.

To demonstrate it numerically, we consider a cylindrical plasma column which has only one-dimensional radial distribution. An equilibrium solution (well-known as the Z pinch) may exist by imposing a constant electric field (E_{z0}) along the axial (z) direction. The corresponding XMHD equations in the cylindrical coordinates are reduced to a nonlinear ordinary differential equation. This order is increased by 2 due to the electron-inertia effect, indicating that it is a kind of singular perturbation for MHD. In this symmetric geometry, the Hall term becomes automatically zero and has no impact on the equilibrium solution.

Since the cylindrical plasma is pinched by electromagnetic force, the plasma region is surrounded by vacuum at the equilibrium state if the plasma beta is sufficiently low. The density profile ρ of the MHD solution is shown by the dashed line in Fig. 1. In XMHD, the density becomes exactly zero at a position $r = a$ if adiabatic change is assumed with a specific heat ratio $\gamma > 1$. The XMHD solution is not smooth at $r = a$ and the finite difference scheme fails there. This singularity needs to be regularized appropriately in numerical analysis. The resultant XMHD solution is shown by the solid lines in Fig. 1, where the resistivity and kinematic viscosity are constant everywhere for simplicity.

The numerical solution indicates that there exists a "boundary layer" that connect the inner MHD solution and the outer vacuum magnetic field. Within the layer, a localized surface electric current is driven due to the electron-inertia effect. Although the collisional effect tends to attenuate this surface current, the constant resistivity is insufficient for that and, hence, electrons are accelerated within the low-density layer. Because of this boundary layer, the XMHD equilibrium solution does not converge to the MHD one in the limit of zero electric-inertia effect. In particular, the equilibrium position of the plasma-vacuum interface shifts inward depending on the magnitude of surface current.

In XMHD, we need to solve the relative velocity $\boldsymbol{u} = \boldsymbol{v}_i - \boldsymbol{v}_e$ of ion and electron even in the vacuum region. Then, the kinematic viscosity and the no-slip boundary condition are necessary for well-posedness and numerical stability. In the MHD limit, the width δ of the boundary layer tends to zero, but the surface current remains finite. By considering matching conditions at plasma-vacuum interface, we can obtain its position $r = a$ analytically, which depends on only the plasma beta.

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References

[1] K. Kimura and P. J. Morrison, Physics of Plasmas 21, 082101 (2014).

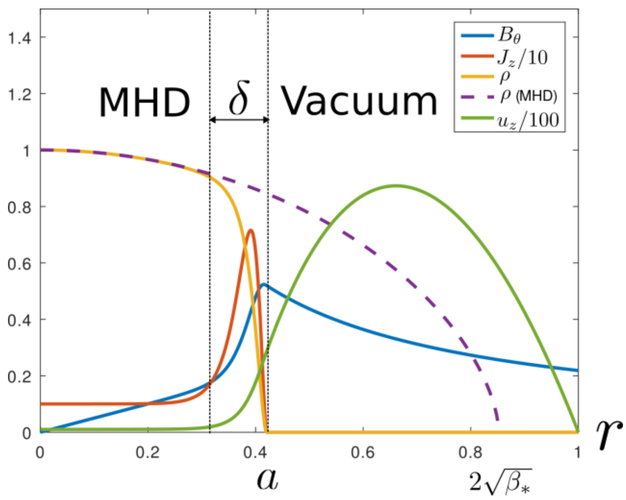

Figure 1. Radial profile of a cylindrical equilibrium of XMHD under axial electric field (Z pinch).