The metriplectic 4-bracket: An inclusive framework for consistently joining Hamiltonian and dissipative systems

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Although an early generalization of Lagrangian mechanics to include dissipation was proposed by Rayleigh (1894) and subsequently various specific frameworks for dissipation have been given, e.g., for phase separation in Cahn-Hilliard (1958) and Ricci flows in Hamilton (1982), here we discuss bracket descriptions for dissipation that were motivated by the noncanonical Poisson bracket formulation of plasma models presented in [1,2] and reviewed in [3]. The motivation of early attempts [3-7] was to place dissipation in a kind of bracket formalism that complements the nondissipative Poisson bracket formalism. The axioms of the formalism were first presented in [6,7] and later in [8] called metriplectic dynamics. Subsequent aspects of metriplectic dynamics were described by the author and collaborators in [9-11].

Here we present the metriplectic 4-bracket [12], an inclusive framework for joined Hamiltonian and dissipative dynamical systems, which preserve energy and produce entropy. The dissipative dynamics of the framework is based on a 4-bracket, a quantity like the Poisson bracket defined on phase space functions, but unlike the Poisson bracket has four slots with symmetries and properties motivated by Riemannian curvature. Metriplectic 4-bracket dynamics is generating using two generators, the Hamiltonian and the entropy, with the entropy being a Casimir of the Hamiltonian part of the system. The formalism includes all known previous binary bracket theories for dissipation or relaxation as special cases. Rich geometrical significance of the formalism and methods for constructing metriplectic 4-brackets are explored. Many examples of both finite and infinite dimensions are given.

We speculate about metriplectic computation. Just as symplectic integrators preserve canonical Hamiltonian structure, with some advantages, there have been recent efforts to preserve noncanonical Hamiltonian structure (see e.g. [13]) and even preserve or use metriplectic structure [14-16]. Another kind of dissipative dynamics, double bracket dynamics, proposed in [17,18] and improved and used in [19-22], provides a method for computation of equilibrium states. This formalism emerges from the metriplectic 4-bracket. To summarize, in this talk I will describe metriplectic 4-bracket dynamics, its construction, reduction to previous brackets, and its possible use for designing computational algorithms.

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