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Linear response function of turbulence and its time scale

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Two-point correlation function, such as the energy spectrum and the Reynolds stress, is the central object in the turbulence research. We can write the equation of a two-point correlation function by using the governing equation, for example, the Navier-Stokes equations. However the resultant equation involves three-point correlation functions. In other words, it is not closed with the two-point correlation functions, known as the closure problem. Therefore we need to model higher order correlation functions somehow in terms of the two-point correlation functions.

One systematic method of such a modeling is arguably to use linear response function. For the closure problem on the energy spectrum of homogeneous isotropic turbulence (HIT), the linear response function was first used by Kraichnan in 1950s [1]. His closure approximation, called direct interaction approximation (DIA), has opened up a number of new research avenues while overcoming its initial failure in predicting the Kolmogorov $k^{-5/3}$ law of the energy spectrum (k is wavenumber). Our first focus here is on one of the avenues, which concerns time scales of the two-point correlation function and, in particular, of the linear response function and how they differ depending on the Eulerian or Lagrangian coordinates. Our second focus is an exact formula of the linear response function, which can be obtained with a recent technique of non-equilibrium statistical mechanics [2].

Regarding the first focus, we perform a direct numerical simulation (DNS) of HIT of a neutral fluid with a forcing in a periodic cube using the spectral method. We then calculate the temporal correlation functions of the Fourier coefficients of the velocity and the linear response function of the same Fourier coefficients. We do this both in the Eulerian and Lagrangian coordinates (for the latter we use so-called Lagrangian history velocity). We find in the inertial range that their time scales in the Eulerian coordinates are proportional to k^{-1} and that the time scales in the Lagrangian coordinates are proportional to $k^{-2/3}$ in agreement with the classical predictions. See Fig.1 for the measured time scale of the linear response function as a function of the wavenumber in both the Eulerian and Lagrangian cases.

Regarding the second focus, we adapt the theory for nonlinear Langevin equations in [2] to HIT by adding a Gaussian random noise term to the Navier-Stokes equation in the Fourier space in addition to the large-scale forcing. We obtain an exact formula of the linear response function of the Fourier coefficients of the velocity in the Eulerian coordinates. It involves two-point and three-point correlation functions and the amplitude of the Gaussian noise. We show that, for sufficiently small noise, the formula agrees with the linear response function without the noise. Also the formula shed some light on the discrepancy between the two-point correlation function and the linear response function (breakdown of the fluctuation dissipation relation).

Further details of this presentation can be found in [3].

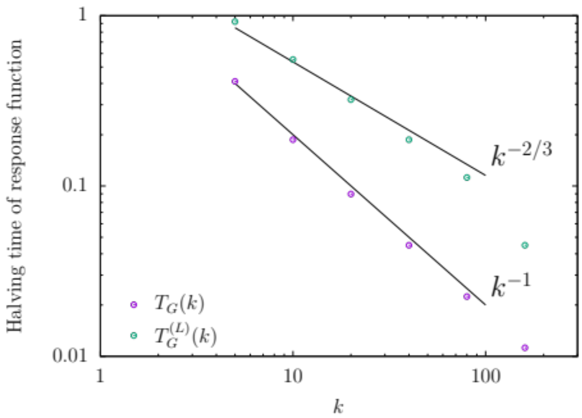

Figure 1. Time scales of the linear response function as a function of wavenumber k. Here the time scale is defined as the halving time of the linear response function (the time at which the linear response function is equal to 0.5). Here $T_G(k)$ corresponds to the time scale of the linear response function of the velocity Fourier coefficients in the Eulerian coordinates. And $T_G^{(L)}(k)$ corresponds to the time scale of the linear response function in the Lagrangian coordinates.

References

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