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Measuring permutation entropy and statistical complexity in plasma

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As a method based on probability information theory, the analysis involving permutation entropy and statistical complexity is becoming widely used in scientific researches [1]. These parameters are useful to categorize time-series data. Complexity is a measure of the interplay between the information stored in the system and its disequilibrium. To evaluate probability distributions, the permutation entropy is applied to time series. Plenty of research have gone into developing this measure, and those have presented distinction between stochasticity and chaos using scatter plots of the entropy versus complexity, often referred to as the complexity–entropy causality plane (or *C–H* plane). Recently, the analytical method has spread to plasma physics researches [2, 3].

Considering a time-series data as a set of tuples $x_i \in \{x_1, \dots, x_N\}$, each tuple is sorted into a set of permutations $y_i \in \{y_1, \dots, y_N\}$. y_i is one of the permutations of x_i. There are K permutations, where K is a factorial of the embedding dimension, $d!$. To determine the probability associated with each y_i, the frequency of each permutation is counted in a time window. The probability for y_i is given as $p_j = \#\{y_j\}/N$ denoting the frequency occurrence of y_j in N tuples. Therefore, the probability distribution is presented as $p_j \in P = \{p_1, \dots, p_K\}$ so that Shannon's information entropy is given as $S = -\sum_{j=1}^{K} p_j \ln p_j$. The normalized entropy can be written as $H_\mathrm{S} = S(P)/S_\mathrm{max}$, which is called permutation entropy. Here, $S_\mathrm{max} = S[P_\mathrm{e}] = \ln K$ with $P_\mathrm{e} = \{1/K, 1/K, \dots, 1/K\}$. Then, Jensen-Shannon statistical complexity is given as $C_\mathrm{JS}(P) = Q_\mathrm{J}[P, P_\mathrm{e}] H_\mathrm{S}(P)$, where $Q_\mathrm{J}[P, P_\mathrm{e}]$ is the disequilibrium between the probabilities.

The analysis with $(H_\mathrm{S}, C_\mathrm{JS})$ was applied to the time-series in the reversed field pinch (RFP) plasma [4]. The data was obtained in an RFP device, RELAX. The typical discharge duration is about 2-3 ms. Fig.1 presents a *C-H* plane scatter diagram for soft X-ray (SXR) emissions and peripheral magnetic fluctuations. These parameters were measured in the conditions with high and low electron density. The number of data points to obtain a probability distribution is 151, and $d = 4$ is selected. Comparing SXR emission features, H_SXR and C_SXR of the high-density regime are clearly lower than those of the low-density regime. This indicates that H_SXR and C_SXR have some dependences on the parameters of the plasma like electron density. Magnetic fluctuation in these two regimes also behave differently. Fluctuations at high-density condition approach the central area of the *C–H* plane but those at low density remain on the fractional Brownian motion curve around $H_\mathrm{SXR} \approx 0.9$. The statistical complexities of the SXR emissions and the magnetic fluctuations also depend on the relationship between reversal and pinch parameter in the RFP experiment. Similar analyses can be applied to time-series data in plasma experiments or even in numerical simulation data.

In addition to the analysis of time-series data, two dimensional images are also characterized by H_S and C_JS. Considering an image as a set of two-dimensional data array, permutations can be formed for tuples consisting of, for example, 2×3 pixels. This analysis gives quantitative measures of complexity of the images. As a test of this method, numerically generated images are analyzed.

Plasma provides structures with complexity through multiple physical phenomena. Even if the mechanism is unknown, two-dimensional structures in plasma can be evaluated by the analysis with H_S and C_JS.

References

[1] O.A. Rosso *et al*., Phys. Rev. Lett. 99 154102 (2007)

[2] W. Gekelman *et al*., Plasma Phys. Control. Fusion 55 064002 (2014)

[3] J. E. Maggs and G.J. Morales, Plasma Phys. Control. Fusion 55 850015 (2013)

[4] T. Onchi *et al*., Phys. Scr. 92 055601 (2017)

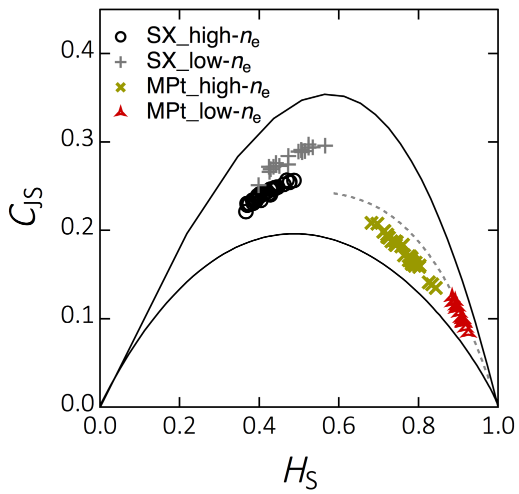

Figure 1. Scatter plots in the *C-H* plane for SXR emissions and peripheral magnetic fluctuations [4].