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Quasi Relaxed Magnetohydrodynamics – phase-space action with $-\nabla_\perp\Phi + \mathbf{u}\times\mathbf{B} = \mathbf{0}$ constraint

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A new formulation of time-dependent Relaxed Magnetohydrodynamics (RxMHD) has recently [1] been derived variationally from Hamilton's Action Principle using a non-canonical phase-space version of the MHD Lagrangian with the phase space variables both being velocity fields, $\mathbf{u}$ and $\mathbf{v}$ (actual and reference flows, respectively, Fig. 1). In the static case, this formalism gives Euler–Lagrange equations consistent with previous work on exact ideal and relaxed axisymmetric MHD equilibria with flow, but generalizes the relaxation concept from statics to dynamics. The new dynamical formalism agrees with ideal-MHD equilibrium theory in the case of flow purely parallel to the magnetic field, i.e. in the *fully relaxed* case when the gradient of the electrostatic field is zero. While the ideal (zero resistivity, no turbulent dynamo) Ohm's Law is not built in and can be shown to be violated in dynamical evolution, the phase space Lagrangian method is shown to be sufficiently flexible as to allow the projection of the electrostatic ideal Ohm's Law, $-\nabla\Phi + \mathbf{u}\times\mathbf{B} = \mathbf{0}$ onto the plane perpendicular to $\mathbf{B}$ to be added as a constraint. This is equivalent to the ansatz

$$\mathbf{u} = \mathbf{B}\times\frac{\nabla\Phi}{B^2} + u_\parallel\frac{\mathbf{B}}{B},$$

which is more constrained than the RxMHD model presented in [1], where $\mathbf{u}$ was varied freely in Hamilton's Principle. Thus it may be termed *Quasi Relaxed MHD* (QRxMHD).

This presentation will present the derivation of the Euler–Lagrange equations for QRxMHD and some of their implications for possible further development of the SPEC code [2] to treat dynamical problems in stellarators as part of the Simons Collaboration on *Hidden Symmetries and Fusion Energy* [3].

References

1. R.L. Dewar, J.W. Burby, Z.S. Qu, N. Sato, and M.J. Hole, Phys. Plasmas **27**, 062504 (2020)
2. S.R. Hudson *et al.*, Phys. Plasmas **19**, 112502 (2012)
3. See https://hiddensymmetries.princeton.edu .

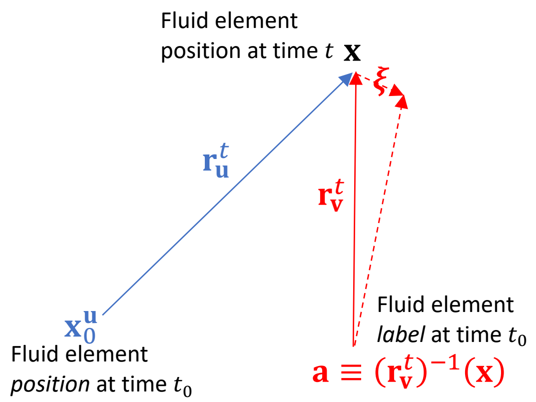

Figure 1. Showing how the inverse of the Lagrangian evolution map $\mathbf{r}_\mathbf{v}^t(\cdot)$ for reference flow $\mathbf{v}$, when acting on points $\mathbf{x}$ in the fluid flow $\mathbf{u}$, provide labels $\mathbf{a}$ alternative to the usual Lagrangian labels $\mathbf{x}_0$. Unlike the $\mathbf{x}_0$, the labels $\mathbf{a}$ evolve in time [1]. However, both labels are fixed when making variations about points $\mathbf{x}$ through displacements $\boldsymbol{\xi}$ at any given time t.