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Time Evolution of Information Entropy in the Landau Damping Process

Koki Maekaku[1], Hideo Sugama[2], Tomo-Hiko Watanabe[3]
[1] Department of Advanced Energy, The University of Tokyo, [2] National Institute for Fusion Science, [3]Department of Science, Nagoya University
e-mail : maekaku.koki20@ae.k.u-tokyo.ac.jp.

We consider a system governed by the Vlasov equation for the distribution function $f(x, v, t)$ of electrons in the two-dimensional phase space (x, v) at time t, and the Poisson equation for the electric field $E(x, t)$ (with periodic boundary conditions of length L in the x-direction). We investigate the time evolution of this Vlasov-Poisson system using the initial condition $f(x, v, t = 0) = f_0(v)\,[\,1 + \alpha\cos(k\,x)]$ where $f_0(v) = n_0\big(\sqrt{2\pi}\,v_t\big)^{-1}\exp\,(-\,v\text{^}2/\,2\,v_t^2\,)$ and $v_t = \left(\frac{T}{m}\right)^{1/2}$ are the Maxwellian distirubion function and the thermal velocity, respectively. Linear and quasilinear analytic solutions describing the short- and long-time behaviors of the distribution function and electric field in the Vlasov-Poisson system were derived and compared with the results of numerical simulations based on Contour-Dynamics [1], and well agreement was confirmed[2].

The linear analytic solutions for the electric field and distribution function exhibiting Landau damping can be represented using an infinite number of complex eigenfrequencies (each with different damping rates) given by the zeros of the dielectric function. By applying appropriate approximations on arbitrary time scales, we derived a quasilinear analytic solution describing the time evolution of the spatially averaged velocity distribution function. We confirmed good agreement with the results of contour dynamics simulations (Fig.1). Furthermore, we accurately determined the limit of the quasilinear distribution function $\langle\, f\,\rangle = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dx}{L}f(x, v, t)$ as $t \to \infty$, calculated its kurtosis, and evaluated the deviation from a Gaussian distribution.

Next, we consider the position x and velocity v of electrons as random variables. By defining the probability distribution at time t as

$$p(x, v, t) = \frac{f(x, v, t)}{n_0 L}$$

where n_0 is the electron density , the normalization condition $\int_{-\frac{L}{2}}^{\frac{L}{2}}dx\int_{-\infty}^{+\infty}dv\,p(x, v, t) = 1$ is satisfied. In the Vlasov system, the information entropy

$$S(X, V) = -\int_{-\frac{L}{2}}^{\frac{L}{2}}dx\int_{-\infty}^{+\infty}dv\,p(x, v, t)\log p(x, v, t)$$

defined from $p(x, v, t)$ is a time-invariant quantity. However, the information entropy

$$S(X) = -\int_{-\frac{L}{2}}^{\frac{L}{2}}dx\,p_X(x, t)\log p_X(x, t)$$

$$S(V) = -\int_{-\infty}^{+\infty}dv\,p_V(v, t)\log p_V(v, t)$$

defined from the marginal distributions $p_X(x, t) = \int_{-\infty}^{+\infty}dv\,p(x, v, t)$ and $p_V(v, t) = \int_{-\frac{L}{2}}^{\frac{L}{2}}dx\,p(x, v, t)$, respectively, and the mutual information $I(X, V) = S(X, V) - S(X) - S(V)$ change over time.

By using the velocity distribution functions from the linear and quasilinear analytic solutions mentioned above, it is possible to accurately calculate the time evolution of the information entropies $S(X)$ and $S(V)$, and the mutual information due to the increase in kinetic energy of particles resulting from Landau damping.

References

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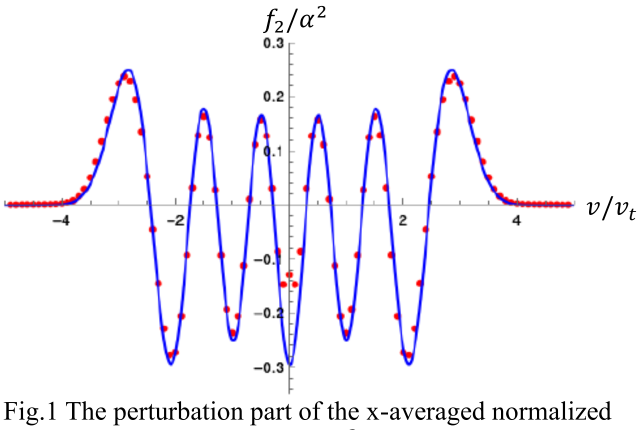

Fig.1 The perturbation part of the x-averaged normalized distribution function $f_2(v,t)/\alpha^2$ is plotted as a function of v/v_t where $f_2(v,t) = \langle f\rangle(v,t) - f_{0M}(v)$ at $t = 10\omega_p^{-1}$, and the red dots represent the result of Contour-Dynamics simulation.