

## Linear equations for stellarator local MHD equilibria around irrational and rational flux surfaces

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Global MHD models provide the magnetic field within a volume enclosed by a flux surface given the shape of that flux surface and the radial profile of pressure and rotational transform. In contrast, local MHD equilibrium equations determine the magnetic field and its radial derivatives on a flux surface of interest given the flux surface shape, the shape of the contiguous flux surfaces and several flux functions. These local MHD equilibria are useful for transport studies (both neoclassical and turbulent) because they can be used to determine how changes in the flux surface shape affect fluxes across that flux surface. In the case of stellarators, local MHD equilibria can also be used to study rational flux surfaces, where the MHD equilibrium equations are expected to fail.

Building on previous work [1, 2, 3], we develop a new set of linear equations to determine the magnetic geometry coefficients needed for local gyrokinetic simulations on a flux surface of interest. Unlike previous local MHD equilibrium models, our equations are linear because they are derived for general angular coordinates. We show explicitly that the final results are independent of the choice of angular coordinates, as one would expect.

Most neoclassical and turbulent transport models only need the magnetic field and its radial derivative on the flux surface of interest. The inputs required to determine the

magnetic field and its radial derivative are the shape of the flux surface, the radial derivative of that shape and four constants. One possible choice for these four constants is the pressure gradient, the gradient of the toroidal flux, and the rotational transform and its radial derivative at the flux surface of interest. Higher order radial derivatives of the magnetic field require higher order derivatives of the pressure and the rotational transform.

When we apply our equations to rational flux surfaces, we find that, for rational flux surfaces to exist, two conditions must be satisfied. One of the conditions is the well-known Hamada condition [4], but the other has not been discussed in the literature to our knowledge. This new condition arises when one calculates the second order radial derivative of the magnetic field, and it constrains the value of the flux surface curvature on rational flux surfaces.

### References

- [1] C.C. Hegna, *Phys. Plasmas* **7**, 3921 (2000).
- [2] A.H. Boozer, *Phys. Plasmas* **9**, 3726 (2002).
- [3] J. Candy and E.A. Belli, *J. Plasma Phys.* **81**, 905810323 (2015).
- [4] S. Hamada, *Nucl. Fusion* **2**, 23 (1962).