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# Description of magnetic field lines without arcana

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Magnetic field lines (B-lines) strongly constrain the dynamics of charged particles, especially if they wrap on nested toroidal magnetic surfaces, as usually done in fusion devices. This fact has generated a huge literature about these lines to describe both the case where they lie on such surfaces, providing good confinement, and the case where they do not, bringing magnetic chaos and a related transport. Unfortunately, a natural tendency for people interested in B-lines in a given device, is to describe them by using directly the magnetic field, instead of the vector potential, that permits to deal with the magnetic field line problem in the frame of Hamiltonian mechanics. Even more unfortunate, overlooking the genuine Hamiltonian character of these lines, often led to wrong results (8 wrong papers quoted in [1] and 6 in [2]). Using the variational principle for magnetic field lines introduced in 1983 by Cary and Littlejohn, this work [3] shows that working with the vector potential is very efficient and simpler than working with the magnetic field.

An innovative pedestrian, yet rigorous, calculation shows this principle can be proved from Stokes theorem applied to the circulation of the vector potential, without other analytical calculations. The action principles for magnetic field lines and for Hamiltonian mechanics are recalled to be analogous. Not only the action principles are analogous, but also a change of canonical coordinates is recalled to be equivalent to a change of gauge. Furthermore, using the vector potential makes obvious the freedom in the choice of "time" for describing Hamiltonian dynamics. These features may be used for a new pedagogical and intuitive introduction to Hamiltonian mechanics.

In the context of confined magnetic fields, the action principle for magnetic field lines makes practical calculations simpler and safer, with an intuitive background and allowing keeping a high degree of generality. This is shown in the practical example of the calculation of the width of a magnetic island, analytically derived without any need of abstract Fourier components, and independently of the choice of coordinates, but by using the magnetic flux through a ribbon whose edges are the field lines related to the O and X point of the island (figure 1). Moreover, a new formula provides explicitly the Boozer and Hamada magnetic coordinates from action-angle coordinates.

*This abstract is proposed for a poster in parallel to an invited talk with the same contents*

References
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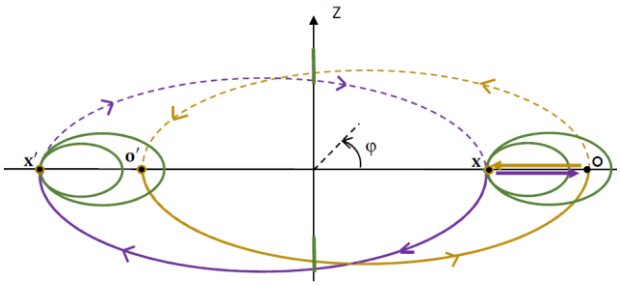


Figure 1