

Machine Learning of Geostrophic Turbulence

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Geostrophic turbulence refers to turbulence in rapidly rotating and stratified systems, of particular relevance to large-scale and mesoscale motion within the oceanic context. Machine Learning techniques have been increasingly deployed as a possibility to supplement and/or replace the subgrid parameterisations for oceanic models, and there is increasing interest in interpretable and/or physics-informed data-driven models, as well as whether it is in fact possible to leverage such data-driven methods to learning about the underlying dynamical principles from the data. With that in mind and to that end, here we revisit two somewhat more basic problems relating to the use of Machine Learning, namely that of data quality, and the training procedure.

We identify a particular problem in relation to a gauge freedom for data that is normally fed into eddy parameterisations in the geostrophic regime, which we argue to contaminate the training of the model. We provide a fix for the gauge removal, and present evidence that the resulting models do better and certainly no worse in terms of skill, but become more robust to small-scale

features, and empirically demonstrate the data contains more information content. We discuss and present results from methodologies relating to physics-constrained learning, namely that from the method of online learning / temporal unrolling, which bears strong resemblance to methodologies in variational data-assimilation, and highlight various advantages and disadvantages of the chosen training procedure. While one could argue the discussion is irrelevant as long as the machine learning model is skillful, we argue here that data quality and methodologies affect for information is extracted from data, which can have impact on the model skill, but may well be fundamental for interpretability and for obtaining physics-informed models. Further speculations on how to quantify information extraction is discussed if time, leveraging ideas from uncertainty quantification.

[1] F. E. Yan, J. Mak & Y. Wang (2024), *J. Adv. Model. Earth. Syst.*, **16**(2), e2023MS003915

[2] F. E. Yan, H. Frezat, J. Le Sommer, J. Mak & K. Otness (in prep.), *J. Adv. Model. Earth. Syst.*

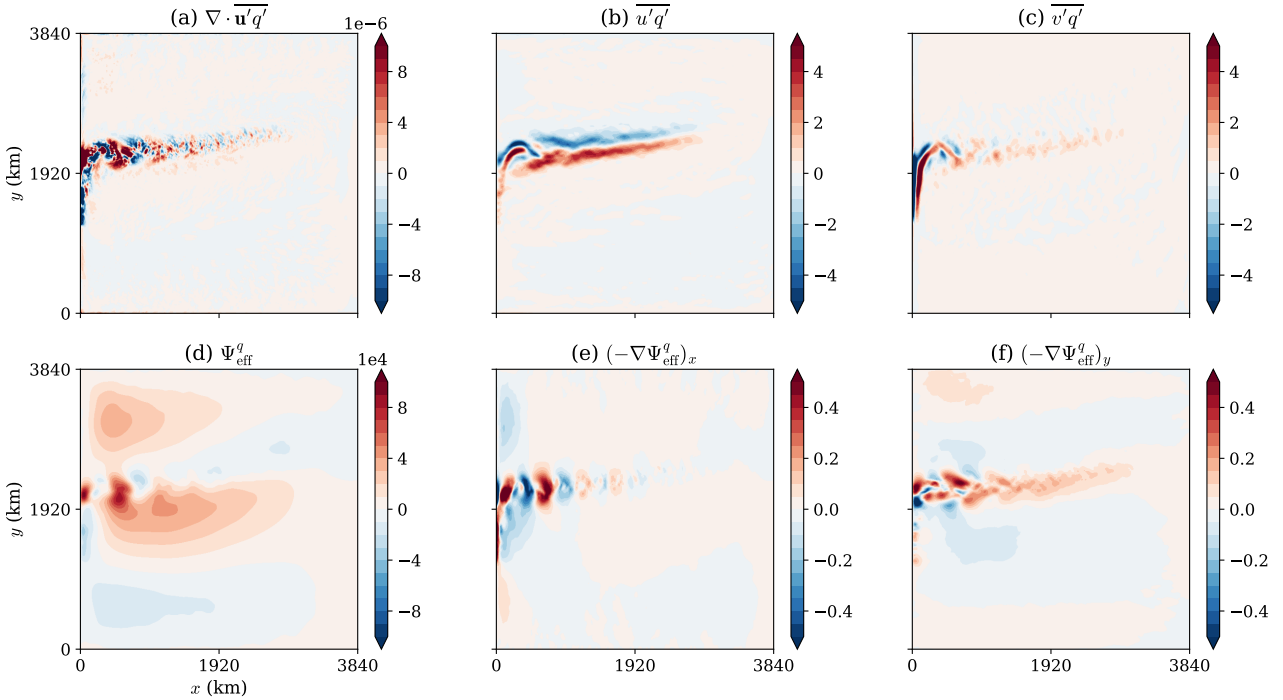


Fig. 1: (a) Divergence of eddy fluxes and (b,c) components of the eddy fluxes. An elliptic solve with appropriate boundary conditions lead to the (d) eddy force function, from which the components of the gradient of the eddy force function (e,f) are the eddy fluxes with rotational gauge removed. Divergence of (b,c) and (e,f) are equal up to numerical discretisation errors.