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A Collision Operator For Describing Dissipation in Noncanonical Phase Space

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This work addresses a fundamental problem in kinetic theory: determining the expression of the collision term in the Boltzmann equation when the underlying phase space is noncanonical. In this study, we derive the collision operator for an ensemble of particles governed by noncanonical equations of motion, showing that the collision term explicitly depends on the Poisson tensor that encodes the noncanonical phase space structure. This dependence alters the nature of thermodynamic equilibria, leading to deviations from Maxwell-Boltzmann statistics and the emergence of self-organizing behavior. Our findings have broad implications for statistical mechanics and plasma physics, with potential applications including collisionless relaxation, self-organization, and inviscid damping in magnetized plasmas and self-gravitating systems.

The phase space of a noncanonical Hamiltonian system is partially restricted due to dynamical constraints (Casimir invariants) arising from the Poisson tensor's kernel. The mechanism by which deviation from Maxwell-Boltzmann statistics occurs is rooted in the time scale over which conservation of Casimir invariants is not broken by particle collisions. Indeed, when an ensemble of such systems interacts, dissipative processes eventually break these constraints, resulting in a Maxwell-Boltzmann equilibrium. However, the timescale to reach this type of thermodynamic equilibrium is often much longer than the timescale for achieving a state of equilibrium. Examples include diffusion in rigid mechanical systems and collisionless relaxation in magnetized plasmas and stellar systems, where binary Coulomb or gravitational collision intervals exceed the timescale for stable structure formation.

This talk focuses on self-organizing phenomena over spacetime scales where particle interactions respect the noncanonical Hamiltonian structure while driving the system towards thermodynamic equilibrium. We derive a collision operator for general noncanonical Hamiltonian systems, applicable to fast, localized interactions. This operator depends on the interactions between colliding particles and the Poisson tensor, ensuring consistency with entropy growth, particle number conservation, energy conservation, and preservation of interior Casimir invariants. It reduces to the Landau collision operator in the limit of grazing binary Coulomb collisions in canonical phase space and exhibits a metriplectic structure. We demonstrate how thermodynamic equilibria deviate from Maxwell-Boltzmann statistics due to the noncanonical phase space structure and describe self-organization and collisionless relaxation in magnetized plasmas and stellar systems using the derived collision operator.

References

[1] Naoki Sato and Philip J. Morrison, "A Collision Operator for Describing Dissipation in Noncanonical Phase Space," Fundamental Plasma Physics, 10, 100054, 2024.