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A Hamiltonian structure-preserving discretization of Maxwell's equations in nonlinear media

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Many asymptotic models describing the interaction of light and matter are characterized by a self-consistent (field dependent) nonlinear polarization and magnetization. For example, a model relevant in plasma physics is the ponderomotive force which, when considered self-consistently, induces a reciprocal density dependent index of refraction [1]. General models describing the constitutive relations for Maxwell's equations in self-consistently polarized and magnetized media may be described by a single, elegant Hamiltonian formalism [2]. This Hamiltonian formalism elucidates the connection between the electromagnetic energy and constitutive relations in nonlinearly polarized electromagnetic media, and facilitates the building of more complex models which couples nonlinear electrodynamics with dynamical models for the medium.

This Hamiltonian modeling framework may be leveraged to design energy-stable and Gauss-conserving finite element methods to spatially discretize Maxwell's equations in nonlinear media. This approach yields a spatially semi-discretized system of ordinary differential equations which are Hamiltonian and conserve discrete analogs of Gauss's laws. The key to exact conservation of Gauss's laws in the semi-discretized system is the use of a spatial discretization which preserves the de Rham cohomology. To accomplish this, one might use mimetic finite differences or finite element exterior calculus (FEEC). This work utilizes a FEEC method which uses B-splines for interpolation [3]. As the spatially semi-discrete system is Hamiltonian, it may be integrated in time using the wealth of structure-preserving methods for Hamiltonian ODEs (e.g. symplectic or energy conserving methods). In particular, systems amenable to Hamiltonian splitting methods yield a particularly convenient time-integration scheme yielding a simple and efficient structure-preserving fully-discrete scheme.

A useful model problem to test this numerical method is Maxwell's equations in cubicly nonlinear media. Such models frequently appear in nonlinear optics. The method used in this work is both energy-stable, and Gauss-conserving, properties which have not been present together in prior time-domain solvers for nonlinear optics.

This work contributes a robust numerical method for the Maxwell subsystem in complex models coupling the electromagnetic field with matter. The Hamiltonian modeling formalism accommodates a broad class of nonlinear constitutive relations for Maxwell's equations, and the numerical method studied in this work simulates such models while preserving the essential features of the continuous model, i.e. energy conservation and the conservation of Gauss's laws.

[1] William Barham, Yaman Güçlü, Philip J. Morrison, Eric Sonnendrücker; "A self-consistent Hamiltonian model of the ponderomotive force and its structure preserving discretization." *Physics of Plasmas,* 1 January 2024; 31 (1): 013905.

[2] William Barham, *On Hamiltonian Structure Preserving Discretizations of Plasma and Nonlinear Wave Models*. 2024, The University of Texas at Austin, Doctoral dissertation.

[3] Yaman Güçlü, Said Hadjout and Ahmed Ratnani. "PSYDAC: a parallel finite element solver with automatic code generation." (2019).