

Orbit tomography in constants-of-motion phase space and the inclusion of prior information

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To understand and predict the dynamics of fast ions is of crucial importance in fusion devices such as tokamaks and stellarators. The 3.5 MeV alpha-particle born in the D-T fusion reaction, which is to be confined in the plasma in order to sustain the high bulk temperatures, is a fast ion, and external heating mechanisms also create fast ions. A population of fast ions is therefore inevitable. In addition to this, recent studies have shown that fast ions can suppress turbulence in the plasma [1]. However, it is also a well-known fact that fast ions can drive instabilities in the plasma [2] deteriorating the plasma confinement. Thus, fast ions are both inevitable, a blessing and a curse. A key to understanding the fast ion dynamics is to measure the fast-ion distribution function, which unfortunately cannot be done directly. We must resort to indirect measurements by solving an inverse problem. Since the fast-ion distribution function is in general six-dimensional, it is convenient to reduce the dimensionality of the problem by utilizing constants of motion. The fast-ion distribution function in tokamaks can be uniquely expressed in terms of three constants of motion and a binary index [3], such that each triplet of constants of motion with the binary index uniquely determines an entire fast-ion orbit.

In this talk, I will explain how the diagnostic sensitivity in constants-of-motion phase space is quantified by so-called weight functions, and how they can be understood via two conditions; a position-space condition and a velocity-space condition, to gain analytical insight [4]. I then continue to show how this enables the possibility of reconstructing the full

three-dimensional fast-ion orbit distribution function from noisy synthetic diagnostic measurements [5]. Included in this is an explanation of how to modify the Tikhonov regularization of the inverse problem to allow for large gradients in specific regions of phase space while promoting smoothness elsewhere. This is explained both from a Tikhonov regularization point of view and from a diffusion equation point of view with a varying diffusion coefficient [6]. As a practical example, I will show how this is used to make an optimization study of the fast-ion diagnostic setup of the future COMPASS-Upgrade tokamak.

Finally, I will also explain how wave-particle interactions, such as ICRF heating, can be incorporated as additional prior information in the inverse problem in energetic particle phase-space tomography. This is done by promoting smooth distribution functions along characteristic streamlines in phase space defined by the fast-ion transport due to wave-particle interactions, in particular those of ICRF heating.

References

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