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# Magnetic fields versus Hamiltonian systems, symmetries versus flux-surfaces: a mathematical cornucopia

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The layout of magnetic field-lines in a fusion device determines the confinement of particles. Qualitatively, the less chaotic the field-lines, the less heat and particles fusion devices will lose. Fusion research typically assumes that magnetic fields conform to nested toroidal flux-surfaces. The 1960s conjecture by Grad suggests that nested surfaces could only exist if the system is axisymmetric, as in tokamaks. On one hand, rigidity of the solutions to the magneto-hydrostatics (MHS) equations has been demonstrated, whereby configurations featuring three-dimensional flux-surfaces, as desired in stellarators, can only be explained through extreme fine-tuning and optimisation. On the other, families of counter-examples have been found in non-Euclidean geometry. The significance of Grad's conjecture relies on the fact that flux-surfaces give rise to special coordinates in which the field-lines are straight, greatly simplifying the theoretical modelling of transport phenomena in fusion devices. A substantial number of plasma models used to study particle transport, fluid turbulence and non-thermal/kinetic effects inherently depend on straight field-line coordinates. If Grad's conjecture is proven true, many theories and results will have to be revised to accommodate for the intrinsic chaotic nature of magnetic field-lines. Sensitivity and robustness to small perturbations is then an important area of investigation. The origin and consequences of chaos is vital for optimising fusion device performance and ensuring efficient confinement of plasma. Estimating the degree of chaos in magnetic field-lines is difficult without expensive numerical field-line tracing. Analysis proceeds using the tools of Hamiltonian perturbation theory. The KAM theorem is then invoked to assess the persistence of irrational surfaces under small perturbations. However, magnetic fields do not necessarily admit Hamiltonian representations. The presentation will delve into the mathematical intricacies of magnetic fields, the structural properties of solutions to the force-balance equation, the deep relationship between symmetries and nested flux-surfaces, the conditions for identifying field-line equations with non-autonomous Hamiltonian systems, and the main obstacles to proving/invalidating Grad's conjecture.

References

[1] Duignan, Perrella and Pfefferlé. "*Global realisation of magnetic fields as 1.5D Hamiltonian systems.*" (2024).

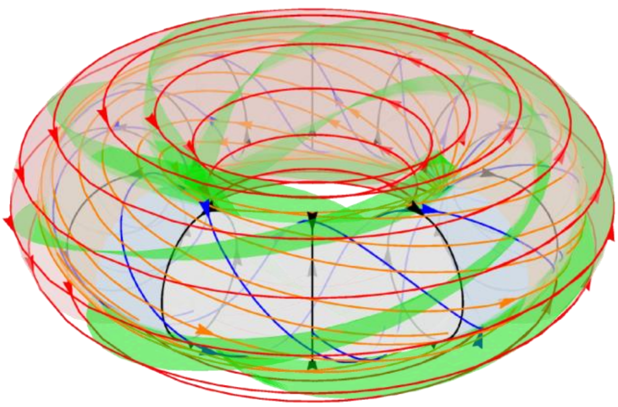

Figure 1: Field-lines and flux-surfaces of a Beltrami field for which the global Poincaré sections are the level-sets of $\phi=\theta+\varphi$ (sum of poloidal and toroidal angle). Poincaré sections are a necessary and sufficient requirement to identify the field-lines as a non-autonomous Hamiltonian system [1].