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Discontinuous codimension-two bifurcation in a Vlasov equation

Yoshiyuki Y. Yamaguchi[1] and Julien Barré[2]
[1] Graduate School of Informatics, Kyoto University,
[2] Institut Denis Poisson, Université d'Orléans, Université de Tours and CNRS
e-mail (speaker): yyama@amp.i.kyoto-u.ac.jp

Vlasov equation describes dynamics of long-range Hamiltonian systems such as plasmas, self-gravitating systems, and two-dimensional Euler fluids. In a spatially periodic Vlasov system, a spatially homogeneous state (velocity distribution) is a stationary solution. By varying a parameter, the coupling strength for instance, the homogeneous state may become linearly unstable. Due to this instability a perturbed unstable state goes away from the initial state and nonlinear effects saturate the amplitude of the instability with the peculiar trapping scaling of square of the instability rate. It is remarkable that the square trapping scaling is much smaller than square root scaling observed in the standard pitchfork bifurcation.

The type of bifurcation depends on the reference velocity distribution however. Let us consider an attractive system. If the homogeneous state is unimodal, symmetric, and smooth, the bifurcation is continuous and the above mentioned trapping scaling is obtained in general. The perfectly flat top of the velocity distribution induces a discontinuous bifurcation instead. Our questions are then: (I) Where is the boundary of flatness to have a discontinuous bifurcation? (II) How can we unify the continuous and discontinuous bifurcations?

We consider smooth, symmetric, and unimodal/bimodal velocity distributions in a spatially one-dimensional attractive system. The answers to the question (I) is that the vanishing second derivative is sufficient to have a discontinuous bifurcation. To the question (II), the two types of bifurcation are unified through the codimension-two bifurcation.

A codimension-two bifurcation is obtained if we tune the values of two parameters. One simple example is the pitchfork bifurcation of the system dx/dt=f(x) with f(0)=0, since f'(0)=f''(0)=0 at the bifurcation point. Another physically important example is a tricritical point in thermodynamics, which is also observed in a Vlasov system [1]. This tricritical point is obtained for a family of so-called waterbag initial distribution, which is a two-valued (a box type) distribution. We underline that our codimensino-two bifurcation is obtained for a smooth distribution.

In our codimension-two bifurcation [2], one of the two parameters to be tuned is the coupling strength, or equivalently the instability rate. It controls stability of the reference homogeneous state. The other parameter is the second derivative of the velocity distribution at the top. The distribution is unimodal (bimodal) if the second derivative is negative (positive), and has a flat top for the vanishing second derivative. See Fig.1.

The codimension-two bifurcation point located at the origin of Fig.1 is characterized in the linear and nonlinear levels. In the linear level, we have a collision of a pair of Landau poles on the imaginary axis, while the collision is before or after the criticality otherwise. In the nonlinear level, the bifurcation is discontinuous only for the vanishing second derivative at the top, while it is continuous with the normal square trapping scaling followed by a jump otherwise.

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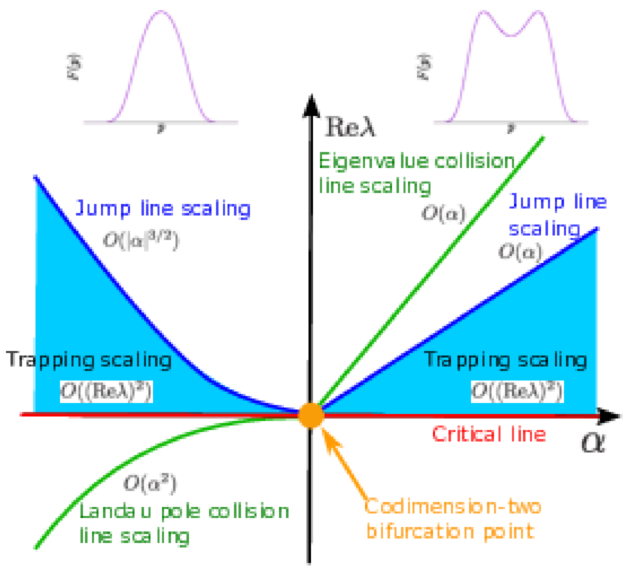

Figure 1. Schematic diagram around a codimension-two bifurcation point. The vertical axis is the instability rate, and the horizontal axis is the second derivative of the velocity distribution function, F''(0). Bifurcation is discontinuous at the codimension-two bifurcation point, and is continuous followed by a jump otherwise.