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A ray-theoretical approach to investigate continuous spectra of two-dimensional incompressible MHD waves on a rotating sphere

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The Alfvén continuous spectrum can appear in an ideal magnetohydrodynamic (MHD) linear wave system in the presence of an inhomogeneous imposed magnetic field. Although the existence of this continuous spectrum has received little attention in the study of planetary interiors, it may play an important role in the dynamics of their liquid cores because of the morphology of their inherent magnetic field.

This presentation focuses on horizontally two-dimensional incompressible MHD waves that propagate within a thin conducting fluid layer over a rotating sphere. This setup can help us understand a stably stratified layer that may exist at the top of the Earth's outer core [1,2]. Such stratified layers may also exist in the interior dynamo region of a planet or a star, including Jupiter, Saturn, and the Sun. As a starting point for understanding the dynamics of these layers, we study ideal MHD linear waves propagating under an axisymmetric toroidal imposed magnetic field whose magnitude depends only on the latitude. Among such background fields, one with the simplest profile is the Malkus field, the magnitude of which is proportional to the sine of the colatitude. Its magnitude can be measured using the Lehnert number $Le = \frac{v_A}{2\Omega R}$, where v_A is the Alfvén speed, Ω is the rotation rate of the sphere, and R is its radius. In the Earth's outer core, we can estimate that $Le \approx 10^{-4}$. In the case of a weak Malkus field (where the Lehnert number is much smaller than unity), two distinct types of waves, fast magnetic Rossby (MR) and slow MR waves, exist as discrete eigenmodes [3,4]. As the Lehnert number increases to the order of unity, these two types of eigenmodes merge into (retrograde and prograde) Alfvén eigenmodes. However, the Malkus field profile may be too simplistic to understand the behavior of waves propagating in planetary interiors. We should therefore assess the influence of the inhomogeneity of imposed fields on these waves.

Here we demonstrate the numerical results of the eigenvalue problem describing the property of eigenmodes under a non-Malkus field [5]. We report that slow MR and Alfvén discrete eigenmodes disappear under a non-Malkus field because they transform into an Alfvén continuous spectrum. The eigenfunctions corresponding to this continuous spectrum indicate that a small Lehnert number reduces their typical meridional wavelengths.

Thus, we can use the ray theory to explain this mode transformation when the Lehnert number is much smaller than unity, as in the Earth's core conditions. Figure 1 illustrates the ray trajectory corresponding to a wave initially satisfying the local dispersion relation of the slow MR wave. This plot shows that the dominant wavenumber of a wave packet evolves during its migration in an inhomogeneous background field and that the mode conversion from slow MR into Alfvén waves can occur. Eventually, the wave packet is absorbed into a critical latitude, at which its azimuthal phase speed is equal to the local Alfvén speed. The transformation of the wave packet explains the disappearance of discrete branches of slow MR waves under non-Malkus fields.

Because slow MR waves in a stably stratified layer at the top of the Earth's core may cause decadal and sub-decadal geomagnetic secular variations, more sophisticated investigations, including the influences of the Ohmic dissipation and the poloidal imposed field, should be conducted in future work.

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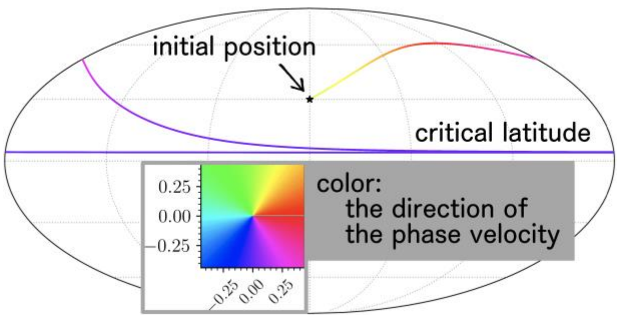

Figure 1. Ray trajectory corresponding to a wave initially satisfying the local dispersion relation of the slow MR wave. The direction of its local phase velocity is shown by hue. This figure is modified from Ref. [5] (CC BY 4.0).