8th Asia-Pacific Conference on Plasma Physics, 3-8 Nov, 2024 at Malacca

Kraichnan's Lagrangian-History Approach to Turbulence: New Insights

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The first instance of a closure approximation without a tuning parameter, which successfully predicts the Kolmogorov energy spectrum $k^{-5/3}$, was introduced by Kraichnan in 1960's [1]. In this closure called ALHDIA, he considered the velocity field in the Lagrangian coordinates, denoted by $\vec{v}(\vec{a}, s|t)$, in addition to the Eulerian velocity field $\vec{u}(\vec{x}, t)$. This Lagrangian velocity corresponds to the velocity of a Lagrangian particle at time t, which passes through coordinates $\vec{a}$ at time s. Usually we consider that the so-called labeling time s is smaller than the so-called measuring time $t (s < t)$ and follow evolution of $\vec{v}$ in terms of the measuring time t. Kraichnan however took the opposite way: he considered $s > t$ and followed evolution in terms of the labeling time s by keeping the measuring time t fixed. This is the Lagrangian history (LH) velocity. The labeling-time evolution is described by the following equations for $\vec{u}(\vec{x}, s)$ and $\vec{v}(\vec{x}, s|t)$:

$$\partial_s \vec{u} + (\vec{u} \cdot \nabla)\vec{u} = -\nabla p + \nu \nabla^2 \vec{u} + \vec{f}, \quad \nabla \cdot \vec{u} = 0,$$

$$\partial_s \vec{v} + (\vec{u} \cdot \nabla)\vec{v} = \vec{0}.$$

Here $\vec{f}$ is a suitable large-scale forcing. Observe that the LH velocity equation is dissipationless and that the LH velocity becomes compressible for $s > t$.

Previous numerical studies on the LH velocity were mostly closure related and certain perculier behaviors were reported [2, 3]. In this study, detaching from closure considerations, we focus on dynamical and statistical behavior of the LH velocity in order to obtain new insights on turbulence. The questions we address includes what shocks in the LH velocity field look like (Figs.1 and 2), how the shocks get steeper in time and what kind of statistical laws the LH velocity has at all [4].

This work is supported by Kakenhi grant 23K03247 from JSPS.

References

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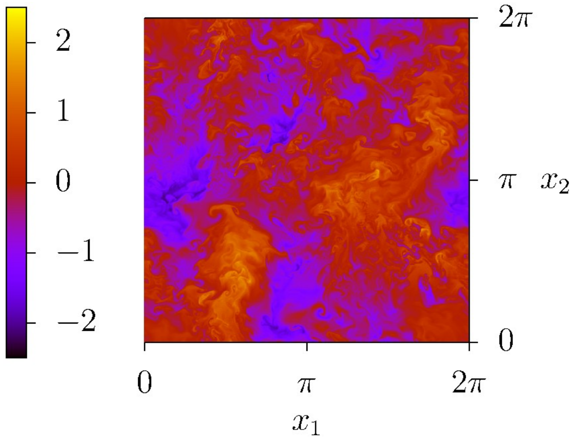

Figure 1. A snapshot of one component of the Lagrangian history velocity. This heat map on a two-dimensional slice ($x_3 = \text{const.}$) shows sharp fronts, which correspond to shocks of the Lagrangian history velocity. The Taylor-microscale based Reynolds number of the background Eulerian velocity is 210.

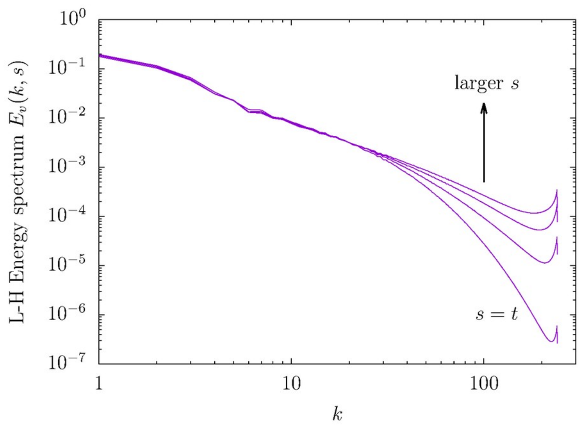

Figure 2. Labeling-time (s) evolution of the Lagrangian history veclocity energy spectrum (see also [2,3]). Notice that the Lagrangian spectrum does not reach a statistically steady state although the Eulerian energy spectrum is in a statistically steady state. The top curve corresponds to the spectrum of the Lagrangian history velocity field shown in Fig.1.